THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION 1972 REVISION

Compiled Under the Authority of the

AMERICAN ASSOCIATION OF WORKERS FOR THE BLIND

ASSOCIATION FOR EDUCATION OF THE VISUALLY HANDICAPPED

and the

NATIONAL BRAILLE ASSOCIATION

Adopted January 20, 1972

by the

AAWB-AEVH-NBA ADVISORY COUNCIL TO THE BRAILLE AUTHORITY

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FOREWORD TO 1972 REVISION

THE NEMETH CODE OF BRAILLE MATHEMATICS AND SCIENTIFIC NOTATION, 1965 initiated sound principles and procedures for the presentation of braille equivalents for the complex signs and configurations of ink-print mathematical and scientific notation. The effectiveness of the Code has been amply demonstrated through its application by transcribers in producing a wealth of technical material to meet the requirements of students at all levels of educational pursuits.

At the time of publication, it was apparent that the Code would require further updating and refinement in order to assure the faithful transference from ink print to braille as new modes of scientific notation were introduced. As was anticipated, problems in interpretation and clarity were encountered when the Code was put into actual use. The comments, criticisms and suggestions from students, teachers and transcribers were taken under consideration in the revision of the Code.

Under the able tutelage of Dr. Abraham Nemeth, the members of the AAWB-AEVH Braille Authority and its Advisory Committee on Mathematical and Scientific Notation entered upon a joint effort in bringing forth a Revised Code which could withstand the test of use and time. As work progressed, however, it became increasingly evident that, because of the complexity of the subject matter and because of the many techniques employed by authors and publishers, substantial research would be required in expanding the Code to its fullest effectiveness. In recognition of this fact, the national Advisory Council to the Braille Authority applied for a planning grant from Social and Rehabilitation Services of the Department of Health, Education, and Welfare. The American Printing House for the Blind was designated as the recipient of the grant, known as the "Braille Codes Pilot Project", which is geared to bring into focus the need for fuller research in all braille codes. Upon the basis of this study, application for a research grant will be made and, if approved, all braille codes will be considered in detail in the endeavor to bring them to maximum completeness and efficiency.

The 1972 revision of THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION provides students and transcribers with a well-drawn, logical system of braille notation which insures a faithful presentation of signs and usages employed in technical texts. The changes which have been incorporated will convey to the reader a realistic picture of the ink-print text and will equip the transcriber with the necessary signs and rules of procedure for a more exact braille transcription.

Grateful acknowledgement is accorded the following persons for their major contribution in the joint effort in developing and refining the revised Code.

Advisory Committee on Mathematical and Scientific Notation

Ralph E. McCracken
Dr. Abraham Nemeth
Mrs. Helen Roberts

AAWB-AEVH Braille Authority

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Freda Henderson (1967-1971)
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THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION 1972 REVISION

ORIENTATION

§1. Description:

a. This Braille Code for Mathematics and Science Notation has been prepared to provide a system of symbols which will allow technical literature to be presented and read in braille. The Code is intended to convey as accurate an impression as is possible to the braille reader of the corresponding printed text, and this is one of its principal features. When the braille reader has a clear conception of the corresponding printed text, the area of communication between himself and his teacher, his colleagues, his associates, and the world at large is greatly broadened. *A test of the accuracy with which the Code conveys information from the print to the braille text is to effect a transcription in the reverse direction. The amount of agreement between the original printed text and one transcribed from the braille is a measure of the Code's accuracy.*

b. A careful distinction is maintained between the meaning which a printed sign has and the sign itself. Sometimes the name of a sign derives from the mathematical meaning which it has. Simple examples are the *plus* sign, the *decimal point*, and the *percent* sign. Other signs have names which are descriptive of the signs themselves, such as *arrow*, *vertical bar*, and *diagonal line*. Still others have names in accordance with the way in which they are read aloud, such as *is less than*, *is contained in*, or *is an element of*. Some signs have no name at all. Of course, the majority of signs, particularly at the lower levels of mathematics, are signs about which there is universal agreement as to their meaning, and these constitute the core about which has grown the modern system of mathematical notation. However, at the more advanced levels, technical writers have, with increasing frequency, been assigning new and unusual meanings to many of the signs which have long been "standard."

c. Although the Code is intended to be as complete as possible, finality can never be achieved by any code. In the course of the rapid development in the fields of science and technology, new signs are constantly being devised and old ones modified. At appropriate places, rules and suggestions are presented for transcribing signs for which no specific provision exists at present.

§2. Organization:

a. In presenting this Code, the needs of both the transcriber and of the reader have been considered. While the rules of the Code have been formulated primarily for the benefit of the braille reader, they are nevertheless presented in a manner designed to ease the transcriber's task of following these rules. The problem of transcription is intrinsically more difficult than the problem of reading; the transcriber must actually recall to mind the specific symbols which must be used and the rules which govern their use, whereas the braille reader must only recognize the symbols which he encounters and be only slightly aware of the underlying rules.

b. This presentation is organized into rules. Where appropriate, each rule begins with a list of signs and their corresponding symbols for quick reference. The body of the presentation is organized into sections which are consecutively numbered and captioned. The sections contain rules, explanations, and examples of the use of the Code. It is intended that the examples be sufficiently definitive so that they may be imitated with confidence in parallel situations. The parenthetical descriptions below the examples are intended to supplement the actual signs shown in the ink print copy. The examples in this presentation are drawn principally from the central core of pure mathematics. Other scientists will find few examples from their fields. Nevertheless, the symbols, rules, and constructions of the basic Code apply with equal force to those fields. Following the rules, there is an INDEX OF BRAILLE SYMBOLS the entries of which have been categorized in accordance with the standard arrangement of the sixty-three braille symbols.

c. Throughout this presentation, the word *sign* is consistently used in referring to a character or sequence of characters in ink print, whereas the word *symbol* is used in referring to a character or sequence of characters in braille.

d. In this text, mathematical or literary material which often appears in italics or other type in ink-print textbooks has been printed in regular type. Italic and other type forms have been used only where such type is required to illustrate a rule.

e. Although 41 cells may be used in transcribing technical works, examples in this text have been shortened to conform to the space available on the ink-print lines.

§3. Interpretation: *It is important that this presentation be accepted quite literally and that no meaning be imputed to the rules and principles which is not expressly stated or directly implied. It may sometimes appear quite arbitrary that a particular sign has been classified in a section which the reader's past experience or training indicates is inappropriate. For the purposes of this Code, however, the transcriber or teacher must accept the classification as well as the rules herewith presented, past experience or technical training notwithstanding.*

In certain situations it may be felt that some constructions are excessively long and there may be a temptation to shorten the construction by the use of a symbol of one's own invention. However, *the transcriber is enjoined against yielding to this temptation.* The Code has been formulated in such a way that the same construction gives the same information to the braille reader from elementary through the most advanced mathematics. Therefore, tampering with the constructions presented herein would have the effect of destroying this uniformity. Signs which for many decades have been exclusively associated with college and graduate mathematics have in recent years been filtering down to high school and grade school levels. Thus, the set operations such as *union, intersection,* and *inclusion,* which were traditionally encountered for the first time by a mathematics major in his junior year in college, have now become fairly commonplace at the fourth or fifth grade level, and are first met even earlier. In addition, grade school and high school mathematics are now being presented with considerably more rigor than heretofore, and shades in meaning are being preserved and even emphasized by the use of distinct signs having similar, but not equivalent, meanings. In keeping with this spirit, the Code furnishes distinct braille symbols corresponding to distinct signs in ink print. In particular, at the lower levels of mathematics, this Code maintains a distinction between the horizontal and diagonal fraction lines, and between the *dot* and the *cross* which signify multiplication. Signs which have separate identities in ink print should be represented by distinct symbols in braille.

§4. Technical and Non-Technical Texts:

a. The designation *non-technical* implies only the absence of mathematical or scientific notation; a work in law or medicine may be quite technical in those fields, but must be regarded as non-technical in the sense just mentioned.

b. *Partially technical* works include science books written for the layman or textbooks in other fields which use mathematical terminology and notation. Such works are characterized by the use of an occasional mathematical sign or a small number of such signs. In works of this kind, the mathematical signs may be treated as in English Braille. This procedure is particularly suitable when there is no intention that the reader should manipulate such signs for the purpose of solving equations or performing computations. Sometimes, however, the replacement of a sign by a corresponding word is not practical, especially when an aggregate of such signs appears in an arrangement which is unusual from the literary point of view. In such cases, the transcriber should use the symbols and the rules of this Code with a note to the braille reader that this is being done. A list of the mathematical symbols being used should be included at the beginning of only the braille volume in which they occur.

c. *Technical* works are those in the fields of mathematics, statistics, physics, or chemistry. In such works the symbols and rules of this Code must be used. They must also be used in works in other fields which make strong use of mathematical signs and modes of expression. In all technical works the transcriber must indicate at the beginning of each volume by means of a transcriber's note that the work has been transcribed in Nemeth Code, giving the year the code was adopted. Even when the Nemeth Code is used, title pages must be transcribed as in English Braille without the use of Nemeth Code symbols, except for items which contain mathematical expressions for which it would be inappropriate or impractical to use English Braille.

d. It is recommended that machines be set for a braille line of 41 cells when transcribing technical works.

RULES

RULE I—BRAILLE INDICATORS

Alphabetic Indicators

English-Letter 

German-Letter 

Greek-Letter

For standard letters 

For alternative letters 

Hebrew-Letter 

Russian-Letter 

Arrow Direction Indicators

Depresses Nearer Arrowhead by 45 Degrees 

Elevates Nearer Arrowhead by 45 Degrees 

Makes Nearer Arrowhead Point Up 

Makes Nearer Arrowhead Point Down 

Arrow Types: Boldface 

Cancellation Indicators

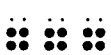
Opening 

Closing 

Capitalization Indicators

Single 

Double 

Carried Number Indicator for Addition
(varying in length) 

Fraction Indicators

Simple

Opening

Closing

Complex

Opening

Closing

Hypercomplex

Opening

Closing

Fractional Part of a Mixed Number

Opening

Closing

General Reference Indicator

Level Indicators

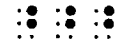
Base Line

Superscript

Superscript with Superscript

Superscript with Subscript

Superscript with Superscript with Superscript



Superscript with Superscript with Subscript

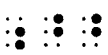
Superscript with Subscript with Superscript

Superscript with Subscript with Subscript

Subscript

Subscript with Superscript

Subscript with Subscript 

Subscript with Superscript with Superscript 

Subscript with Superscript with Subscript 

Subscript with Subscript with Superscript 

Subscript with Subscript with Subscript 

Modification Indicators

Multipurpose 

Directly Over

First order 

Second order 

Directly Under

First order 

Second order 

Superposition 

Termination 

Multipurpose Indicator 

Numeric Indicator 

Punctuation Indicator 

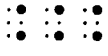
Radical Indicators

Index-of-Radical 

Order-of-Radical

First inner radical 

Second inner radical 

Third inner radical 

Termination 

Shape Indicators

Shape

Structural Shape-Modification

Interior Shape-Modification

Filled-In Shape

Shaded Shape

Termination

Termination Indicator

Type-Form Indicators for Letters, Numerals, and Compound Expressions

Boldface Type

Italic Type

Sanserif Type

Script Type

Type-Form Indicators for Words, Phrases, and Mathematical Statements

Opening Boldface Type

Opening Italic Type

Closing Boldface Type

Closing Italic Type

§5. **Concept of Braille Indicators:** Mathematical expressions are represented in ink print by the use of arbitrary signs among which are the digits, the lower-case and capitalized letters of several alphabets, the script, italic, and boldface forms of these same letters, as well as numerous signs of operation, signs of comparison, signs of grouping, and many other signs serving the miscellaneous requirements of mathematical and scientific expression. Furthermore, mathematical significance is imparted not only by these signs separately, but by their collective arrangement on levels above or below a reference line of writing, as well as by their disposition above or below a fraction line. With only sixty-three distinct braille characters available, sixty-four if the space is counted, the accomplishment of this Code is to make provision for the representation of all these signs, as well as to give an indication of their arrangement.

It is, of course, impossible to establish a one-to-one correspondence between the sixty-three braille characters and the hundreds of signs used in modern mathematics. It is also impractical, as a general procedure, to imitate the arrangement of these signs at various levels relative to a reference line of writing or to a fraction line. Accordingly, the Code presented in the following pages is characterized by the use of a system of braille indicators. The braille indicators in this Code play the same role as do the composition signs

of English Braille. In both systems, the braille indicators or the signs of composition correspond to no sign in ink print; however, they have the power of imparting meaning to the braille symbols with which they are associated. While there are only a few signs of composition in English Braille, there are many in this Code. By their use it is possible to represent the numerous type forms and alphabets used in ink print and to convey the "two-dimensional" information contained in ink print through the medium of the braille system whose nature is essentially "one-dimensional."

§6. Spacing with Braille Indicators: No space should be left between a braille indicator and the symbol or expression to which it applies. In addition, the punctuation indicator, level indicators, and modification indicators apply both to the material which precedes as well as to the material which follows them. There are special spacing rules for the type-form indicators for words, phrases, and mathematical statements (see §33). Examples illustrating this spacing rule are found throughout the Code.

RULE II—NUMERIC SIGNS AND SYMBOLS

Numeric Indicator

Arabic Digits (Nemeth Code)

0	1	2	3	4	5	6	7	8	9
									

Comma (mathematical)

American , 

Continental . 

Decimal Point

American . 

Continental , 

§7. Representation of Arabic Numerals:

a. Digits are represented in two ways: as in English Braille, and as in the Nemeth Code. The digits in English Braille are represented by the letters "j" and "a" through "i". The digits in the Nemeth Code are represented by the symbols whose configurations correspond to these same letters, but which occupy the lower portion of the braille cell.

b. Even when a work is transcribed in the Nemeth Code, when at all practical or appropriate, all numerals on title pages must be transcribed as in English Braille. Numerals at the corners of pages and at the ends of page-separation lines must also be transcribed as in English Braille. English Braille numerals must be used when the technique of "keying" (§187) is employed. In all other cases, including contents pages, forewords, introductions, page references, footnotes, indices, and bibliographies, the numerals of the Nemeth Code must be used.

§8. Comma, Decimal Point:

a. The transcriber should be alert to the possibility that variant forms of the comma and decimal point are sometimes employed, particularly in books published outside of the United States. Although the ink print signs for the comma and decimal point differ from

those used in the United States, this difference is not reflected in the braille transcription. However, a transcriber's note should be included at the beginning of the book to inform the reader of the continental usage in the ink-print edition.

(1) 1,378 

(American usage of comma)

(2) 1.378 

(Continental usage of comma)

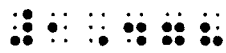
(3) 3.76 

(American usage of decimal)

(4) 3,76 

(Continental usage of decimal)

b. *The comma,* American or Continental, which is interior to a numeral, and which is used to partition the numeral into short regular segments, must be regarded as a numeric symbol. As such, the comma is subject to the rules for transcribing numerals.

(1) 1,478 

(the comma is a numeric symbol, not a punctuation mark)

(2) 100, 200, 300 

(the commas are punctuation marks, not numeric symbols)

c. *The decimal point,* American or Continental, should be regarded as a numeric symbol only when it is associated with a numeral. An omission symbol must not be regarded as a numeric symbol. As a numeric symbol, the decimal point is subject to the rules for transcribing numerals.

(1) .35 

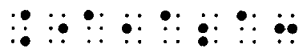
(the decimal point is a numeric symbol)

(2) 3.14 

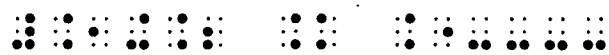
(the decimal point is a numeric symbol)

(3) $.2a_1a_2a_3$ 

(the decimal point is a numeric symbol)

(4) $.a_1a_2a_3$ 

(the decimal point is not a numeric symbol)

(5) $.1 + .2 = .\text{——}$ 

(the first two decimal points are numeric symbols)

§9. Use of the Numeric Indicator: The numeric indicator must be used to introduce one or more unspaced numeric symbols under the following circumstances:

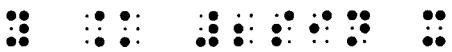
a. The numeric indicator must be used at the beginning of a braille line or after a space. It must also be used after a minus symbol which occurs at the beginning of a braille line or which follows a space. For exceptions, see **§11.**

(1) 27 

(2) There were 7 balls. 

(3) $1 + x + y = 0$ 

(1 plus x plus y equals 0)

(4) $y = 2 \sin x$ 

(y equals 2 sine x)

(5) sin 1 

(6) $\sin^2 2x$ 

(sine squared of 2x)

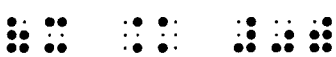
(7) 0.333 ... 3 ... 

(8) $\log_{10} 2$ 

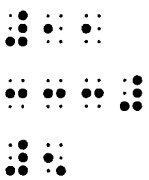
(logarithm to the base 10 of 2)

(9) $\angle 1$ 

(angle 1)

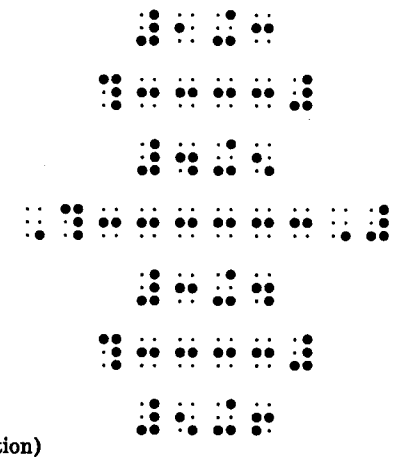
(10) $(x = 0)$ 

(x equals 0 enclosed in parentheses)

(11) $\frac{11}{5}$ 

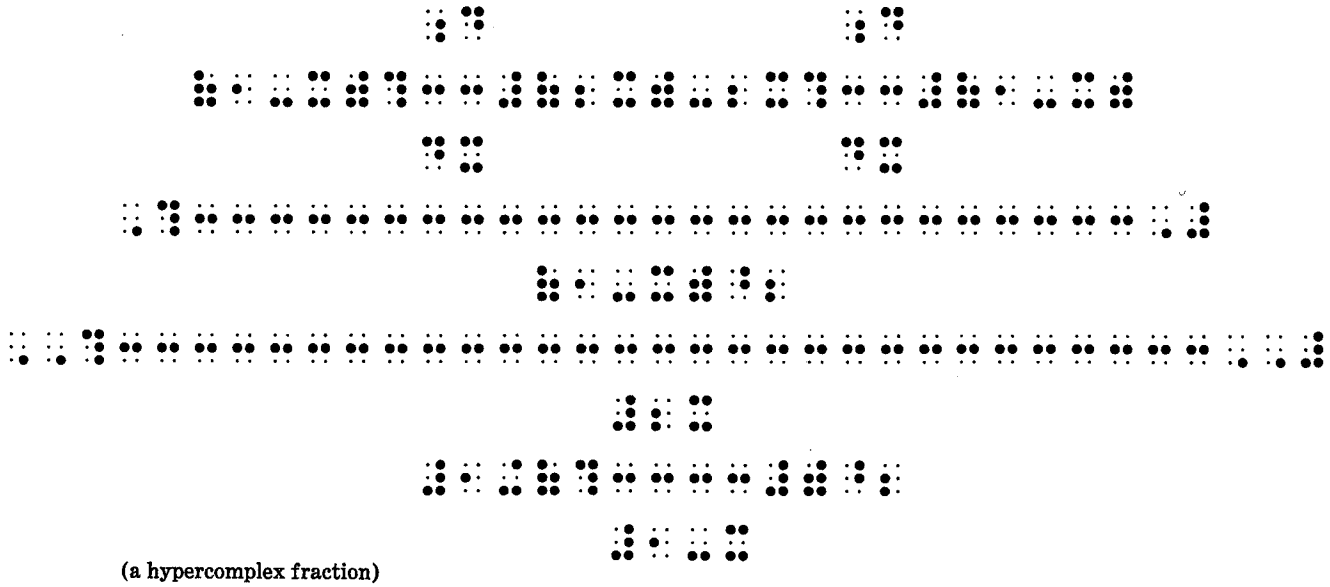
(a simple fraction)

(12) $\dfrac{\dfrac{1+3}{4+5}}{\dfrac{3+4}{5+6}}$



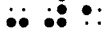
(a complex fraction)

(13) $$\frac{\dfrac{(1-x)\,\dfrac{d}{dx}\,(2x) - 2x\,\dfrac{d}{dx}\,(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$



(a hypercomplex fraction)

(14) -1



(minus 1)

(15) $-.3$

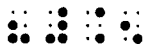
(minus three tenths)

b. The numeric indicator must be used after a punctuation mark. However, the hyphen requires special attention (see section f below). It must also be used after a minus symbol which follows a punctuation mark.

(1) "3 dogs"

(2) Probability——0

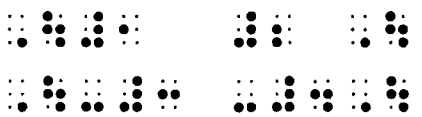
(3) ".5



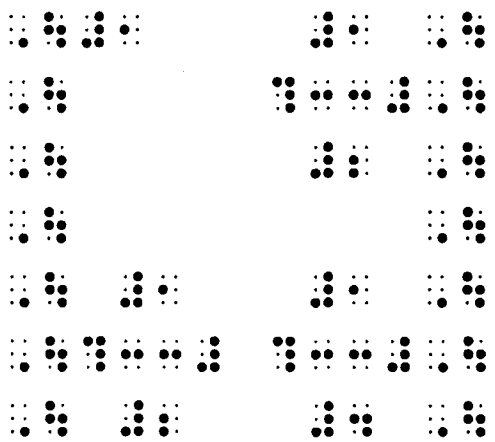
(4) "—4

c. The numeric indicator must be used after a left grouping symbol which introduces a determinant or matrix. It must also be used after a minus symbol which follows such a grouping symbol.

(1) $\begin{vmatrix} 1 & 2 \\ -3 & -4 \end{vmatrix}$



(2) $\begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{vmatrix}$

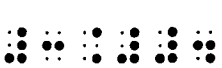


d. The numeric indicator must be used after a section mark, paragraph mark, crosshatch, or asterisk. It must also be used after the general reference indicator or any reference symbol.

(1) 3 § 4

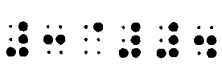
(3 section mark 4)

(2) 3 # 4

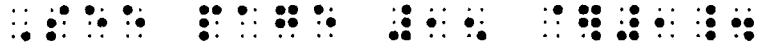


(3 crosshatch 4)

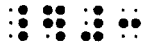
(3) 3 * 4



(3 asterisk 4)

(4) See page 15[1]. 

(1 follows the general reference indicator; the 1 is raised in ink print)

(5) †3 

(dagger 3)

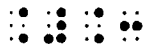
e. The numeric indicator must be used after any of the type-form indicators, or after making a transition from non-regular to regular type within the same numeral. It must also be used after the interior shape-modification indicator.

(1) *3* 

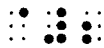
(italicized 3)

(2) 0 

(boldface zero)

(3) .*3* 

(italicized three tenths)

(4) *2* 

(script 2)

(5) **43**56 

(the first two digits in boldface type, the last two in regular type)

(6) ⑤ 

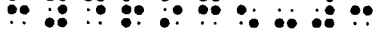
(5 enclosed within a circle)

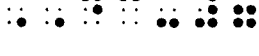
(7) 5 

(5 enclosed within a square)

f. The numeric indicator must be used after a hyphen when the hyphen follows a word, an abreviation, or a mark of punctuation. However, also see §11d.

(1) 1-to-1 correspondence 

(2) hydrogen-3 

(3) DC-7 

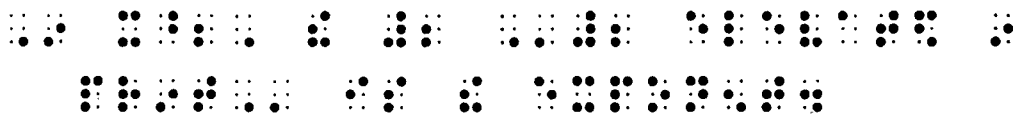
(4) B-49 bomber 

(5) U-238 

(6) (287?-212 B.C.) 

g. The numeric indicator must be used after the opening transcriber's grouping symbol.

(1) In x^2, the ^{2} is the exponent.



§10. Definition of "Enclosed List":

An "enclosed list", for the purposes of this Code, must meet the following requirements:

i. It must begin and end with a sign of grouping. These signs of grouping do not necessarily have to be of the same kind.

ii. It must contain no word, abbreviation, ordinal ending, or plural ending.

iii. A function name, an abbreviated function name, or a sign of shape and the signs which follow them are regarded as a single item.

iv. An item of the list may be the ellipsis or any sign used for omission.

v. No sign of comparison may appear anywhere within the list.

vi. The list must have at least two items. The items of the list must be separated only by commas; the list must not contain any other kind of punctuation mark (except the ellipsis or the long dash which is used for omission) and the space cannot be the sole means for separating items.

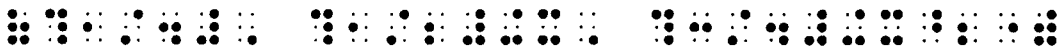
(1) 1, i, —1, —i 

(not an "enclosed list" according to i)

(2) (a, b] 

(an "enclosed list"; meets all requirements)

(3) $(\frac{1}{4}, \frac{1}{2} + x, \frac{3}{4} + x^2)$



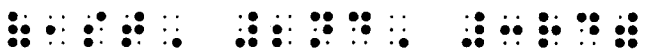
(an "enclosed list"; meets all requirements)

(4) (1, 2, and 3) 

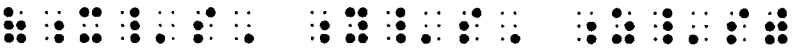
(not an "enclosed list" according to ii)

(5) (h ft, k in) 

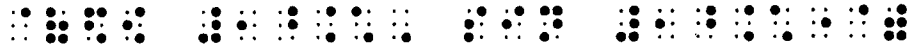
(not an "enclosed list" according to ii)

(6) (1st, 2nd, 3rd) 

(not an "enclosed list" according to ii)

(7) (x's, y's, z's) 

(not an "enclosed list" according to ii)

(8) [$\angle 1°$, sin 1°] 

(angle 1 degree, sine 1 degree; an "enclosed list"; meets all requirements)

(9) (a, b, ...) 

(an "enclosed list"; meets all requirements)

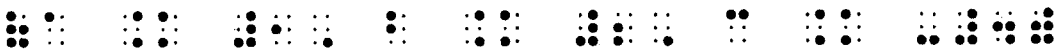
(10) (x + 1, x + 2, ?, ?, x + 5) 

(an "enclosed list"; meets all requirements)

(11) (x = 1, 2, ..., 10) 

(not an "enclosed list" according to v)

(12) (a = 1, b = 2, c = —4)



(not an "enclosed list" according to v)

(13) (u, v; x, y) 

(not an "enclosed list" according to vi)

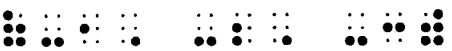
(14) (1 2 3) 

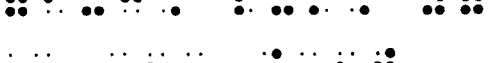
(not an "enclosed list" according to vi)

§11. Non-Use of the Numeric Indicator: It must not be assumed that because a symbol is numeric that the numeric indicator must be used with that symbol. The numeric indicator must not be used preceding a numeric symbol under the following circumstances:

a. The numeric indicator must not be used at the *beginning* of an item which is part of an "enclosed list" as defined in §10 above, even if such an item has been run over to another line. However, if any item in an "enclosed list" is a numeral in a type form other than regular type, that item requires the numeric indicator.

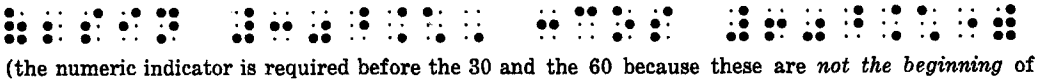
(1) [0, 1] 

(2) (—1, —2, —3) 

(3) (1 + h, 2 + k, 0) 

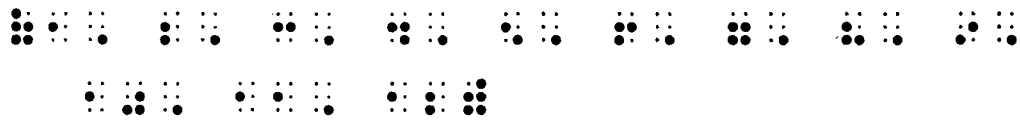
(4) (0, —1, ±2)

(5) (2 sin 30°, 3 cos 60°)



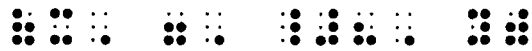
(the numeric indicator is required before the 30 and the 60 because these are *not the beginning* of their respective items)

(6) (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)



(the numeric indicator is not required before 10 even though 10 begins a new line)

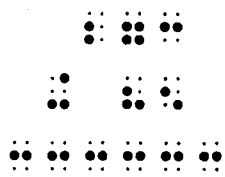
(7) (x, 7, 8, y)



(the 8 in this "enclosed list" is in boldface type)

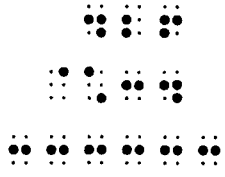
b. The numeric indicator must not be used in work arranged in columns and aligned for addition, subtraction, multiplication, division, or alignment of a system of equations.

(1)
$$\begin{array}{r} 273 \\ +\ 85 \\ \hline \end{array}$$



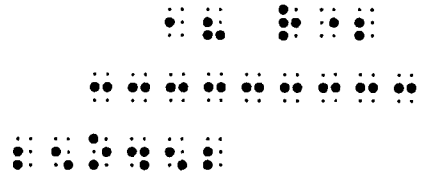
(a problem aligned for addition)

(2)
$$\begin{array}{r} 426 \\ \times\ 34 \\ \hline \end{array}$$



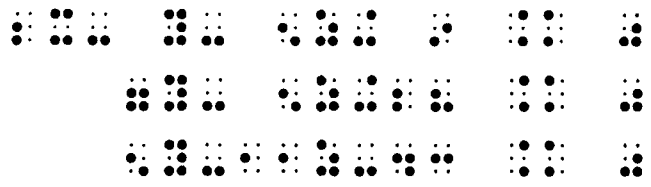
(a problem aligned for multiplication)

(3)
$$25\overline{)452}\quad 18\text{ r }2$$



(a problem aligned for division)

(4)
$$\begin{aligned} 2x - y - 5z + 9 &= 0 \\ 7y - 5z + 28 &= 0 \\ 5y - 11z - 43 &= 0 \end{aligned}$$



(alignment of three equations)

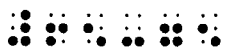
c. The numeric indicator must not be used after a space if the purpose of the space is to partition a numeral into segments.

(1) $\pi = 3.14159\ 26535\ \ldots$

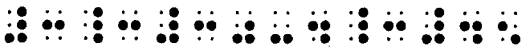
(2) 947, Millions 147, Thousands 592 Ones or Units

d. The numeric indicator must not be used after a hyphen if the hyphen follows a numeral, a letter, or other mathematical expression.

(1) 65-75



(2) 3:30-4:45

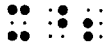


(three thirty to four forty-five)

(3) Read section A-12.

e. The numeric indicator must not be used in any situation not covered in §§9 and 11.

(1) x^2

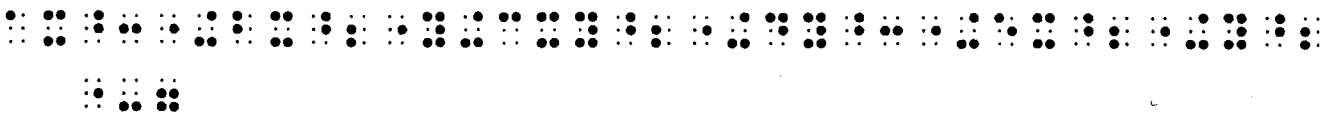


(2) $\frac{3}{x}$

(3) r 5

(remainder of 5 as in a division problem)

(4) $ax^3 + bx^2y + cxy^2 + dy^3 + ex^2 + y^2 - 7$



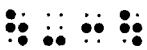
(5) $x - 5$

(6) 2×4

(7) 10,000

(in this numeral, the comma is not a punctuation mark)

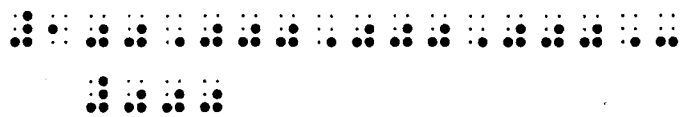
(8) $|-3|$



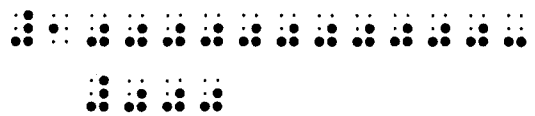
(the absolute value of minus 3)

§12. Long Numerals: Long numerals that cannot be completely accommodated on one braille line may be divided and run over to another line. Such a division must be made after a comma, if present, and a hyphen must be supplied. The numeric indicator must be used as the first braille symbol of the braille line to which the numeral has been run over.

(1) 100,000,000,000,-
000

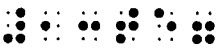


(2) 100000000000-
000

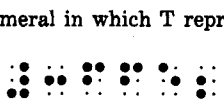


§13. Representation of Numerals to Non-Decimal Bases:

a. When a system of numeration is to a base other than 10, a common technique for providing additional digits is to use letters, either lower-case or capitalized, in addition to the ten Arabic digits. When this technique is used, the transcriber must use only lower-case letters. If capitalized letters are used in ink print, the transcriber must indicate this fact in a transcriber's note.

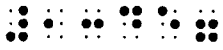
(1) 13TE7 

(a base-12 numeral in which T represents 10 and E represents 11)

(2) 3FFE2 

(a base-16 numeral in which E represents 14 and F represents 15)

b. Another common technique for providing additional digits is to use standard or arbitrary signs to supplement the ten Arabic digits. Authors sometimes give names to these signs. For example, X (dek) represents 10 and Ɛ (el) represents 11. In this case, the transcriber must devise one-cell symbols for these signs, preferably chosen from among the letters of the English alphabet, and must insert a transcriber's note to specify the meanings which have been assigned to these symbols. The transcriber's note must include a drawing of any sign for which there is no equivalent symbol in the Code.

(1) 13XƐ7 

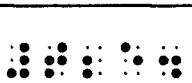
(a base-12 numeral in which X represents 10 and Ɛ represents 11. Here the transcriber has assigned  to X and  to Ɛ)

c. Another common technique is to use an arbitrary set of signs which do not include Arabic digits. In this case, the transcriber must proceed as in **b** above.

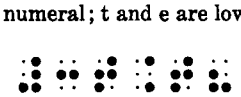
(1) @%$ 

(a base-3 numeral of three digits; here the transcriber has assigned  to @,  to %, and  to $)

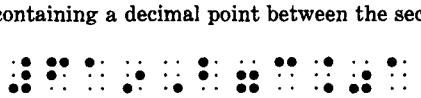
d. The one-cell symbols which the transcriber uses to represent the digits of a non-decimal numeration system must be regarded as numeric symbols. As such, these numeric symbols are subject to the rules for transcribing numerals.

(1) t2e4 

(a base-12 numeral; t and e are lower case in ink print)

(2) 3t.t8 

(a base-12 numeral containing a decimal point between the second and third digits; t is lower case in ink print)

(3) FA9,B7C.0A 

(a base-16 numeral containing a comma and a decimal point; F, A, B and C are capitalized in ink print)

§14. Ordinal Endings: (See §55d.)

§15. Plural and Possessive Endings: (See §39.)

§16. Numerals in Diagrams: In diagrams which contain numeric labels, the numeric indicator must be used. The space to accommodate the numeric indicator may often be gained by a sufficient enlargement of the diagram.

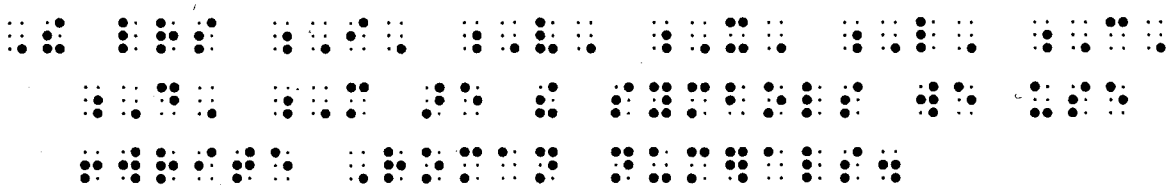
§17. Numerals in Table Entries: In tables whose entries consist entirely of numerals, the numeric indicator must be omitted. However, in tables whose entries are a mixture of words, numerals, letters, or other mathematical signs, the numeric indicator must be used. This rule applies only to the body of a table and not to the headings. Determinants and matrices are not to be regarded as tables. The minus symbol is not numeric so that, if it occurs in a table, the numeric indicator must be used throughout the table.

§18. Roman Numerals:

a. Capitalized Roman numerals must be transcribed using the single capital sign before one letter and the double capital sign before more than one letter. For the use of the English-letter indicator with Roman numerals, see §28c.

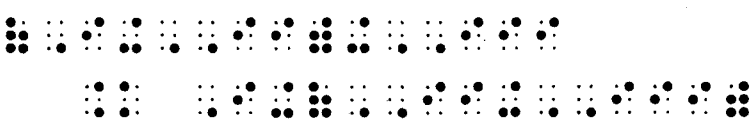
(1) I, II, III, IV, V.

(2) The letters I, V, X, L, C, D, M are the symbols we use to write Roman numerals.



(3) VII + V = XII 

(4) (I + II) + III = I + (II + III)



(5) II′

(prime sign follows the Roman numeral)

(6) $\overline{M}$

(bar over M)

(7) V = 5, L = 50.

(8) Read chapters I-V and XI-XV.

(9) 1.
(I)
(II)

(10) 1.
I.
II.

b. When a Roman numeral consists of one or more lower-case letters it must be treated as though it were a "single letter" and, as such, the English-letter indicator must be used or not used in accordance with the rules governing the English-letter indicator (see §§26-28).

(1) i, ii, iii, iv, v.

(2) See pages v and vi.

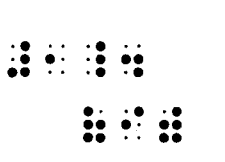
(3) ¶a, §i and §ii.

(4) vi + iv = x

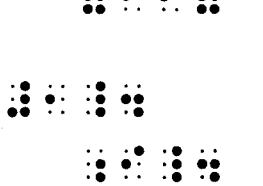
(5) i = 1, v = 5, and x = 10.

(6) Read pages i-v and xi-xv.

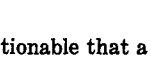
(7) 1.
(i)
(ii)



(8) 1.
i.
ii.



c. When it is questionable that a letter combination is a Roman numeral, treat the combination as if it is not a Roman numeral.

(1) CL 

(this combination occurs in a context from which it cannot be ascertained whether CL is a Roman numeral)

(2) mix 

(this combination occurs in a context from which it cannot be ascertained whether mix is a Roman numeral)

d. For punctuation of Roman numerals, see §37, iii.

§19. **Spacing with Numerals:** Spaces within numerals must be left when it is necessary to partition a numeral into short regular segments, or to achieve alignment. (For examples see §11b and c.)

RULE III—CAPITALIZATION

Capitalization Indicators

Single

Double 

(For combinations of capitalization, alphabetic, and type-form indicators, see Appendix, page 208)

§20. Use of the Capitalization Indicator:

a. The capitalization indicator must be used to indicate the capitalization of a letter from any of the alphabets listed in **Rule IV**, except the Hebrew alphabet whose letters do not possess a capitalized form. This indicator must precede the letter concerned.

(1) 𝔄 

(German capitalized ah)

(2) Γ 

(Greek capitalized gamma)

b. For capitalized Roman numerals, see **§18a.**

c. For the capitalization of abbreviations, see **§50.**

§21. Non-Use of the Capitalization Indicator: Capitalization must not be used with a letter just because it begins a sentence, if the corresponding letter in ink print is uncapitalized.

(1) x is a number between 2 and 3.

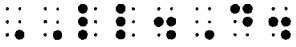
§22. Effectiveness of the Capitalization Indicator:

a. The effectiveness of the single capitalization indicator extends only to the letter which follows it, so that if each letter in a sequence requires capitalization, the capitalization indicator must be used with each of these letters individually.

(1) △ ABC 

(triangle ABC)

b. The effectiveness of the double capitalization indicator in Roman numerals and in abbreviations extends to all of the letters which immediately follow it. However, a symbol other than a letter terminates its effect.

(1) LL.D. 

(2) III + V 

RULE IV—ALPHABETS

Alphabetic Indicators

English (Roman) Letter 

German-Letter 

Greek-Letter

For Standard Letters ⠨

For Alternative Forms of Letters ⠨ ⠨

Hebrew-Letter ⠸ ⠸

Russian (Cyrillic) Letter ⠨ ⠨

(For combinations of capitalization, alphabetic, and type-form indicators, see Appendix, page 208)

Alphabets

English (Roman) Alphabet

Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Sanserif capitalized	Braille equivalent
a	A	𝒶	𝒜	A	⠁
b	B	𝒷	ℬ	B	⠃
c	C	𝒸	𝒞	C	⠉
d	D	𝒹	𝒟	D	⠙
e	E	ℯ	ℰ	E	⠑
f	F	𝒻	ℱ	F	⠋
g	G	ℊ	𝒢	G	⠛
h	H	𝒽	ℋ	H	⠓
i	I	𝒾	ℐ	I	⠊
j	J	𝒿	𝒥	J	⠚
k	K	𝓀	𝒦	K	⠅
l	L	𝓁	ℒ	L	⠇
m	M	𝓂	ℳ	M	⠍
n	N	𝓃	𝒩	N	⠝
o	O	ℴ	𝒪	O	⠕
p	P	𝓅	𝒫	P	⠏
q	Q	𝓆	𝒬	Q	⠟
r	R	𝓇	ℛ	R	⠗
s	S	𝓈	𝒮	S	⠎
t	T	𝓉	𝒯	T	⠞
u	U	𝓊	𝒰	U	⠥
v	V	𝓋	𝒱	V	⠧
w	W	𝓌	𝒲	W	⠺
x	X	𝓍	𝒳	X	⠭
y	Y	𝓎	𝒴	Y	⠽
z	Z	𝓏	𝒵	Z	⠵

German Alphabet

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	𝔞	𝔄			⠁
beh	𝔟	𝔅			⠃
tseh	𝔠	ℭ			⠉
deh	𝔡	𝔇			⠙
eh	𝔢	𝔈			⠑
eff	𝔣	𝔉			⠋
gheh	𝔤	𝔊			⠛
hah	𝔥	ℌ			⠓
ee	𝔦	ℑ			⠊
yaht	𝔧	𝔍			⠚
kah	𝔨	𝔎			⠅
ell	𝔩	𝔏			⠇
em	𝔪	𝔐			⠍

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
en	𝔫	𝔑			⠝
oh	𝔬	𝔒			⠕
peh	𝔭	𝔓			⠏
koo	𝔮	𝔔			⠟
err	𝔯	ℜ			⠗
ess	𝔰	𝔖	or		⠎
teh	𝔱	𝔗			⠞
oo	𝔲	𝔘			⠥
fao	𝔳	𝔙			⠧
veh	𝔴	𝔚			⠺
iks	𝔵	𝔛			⠭
ypsilon	𝔶	𝔜			⠽
tset	𝔷	ℨ			⠵

Greek Alphabet (Standard)

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
alpha	α	A			⠁
beta	β	B			⠃
gamma	γ	Γ			⠛
delta	δ	Δ			⠙
epsilon	ϵ	E			⠑

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
zeta	ζ	Z			⠵
eta	η	H			⠱
theta	θ	⊛			⠹
iota	ι	I			⠊
kappa	κ	K			⠅

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
lambda	λ	Λ			
mu	μ	M			
nu	ν	N			
xi	ξ	Ξ			
omicron	ο	O			
pi	π	Π			
rho	ρ	P			

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
sigma	σ	Σ			
tau	τ	T			
upsilon	υ	Y			
phi	φ	Φ			
chi	χ	X			
psi	ψ	Ψ			
omega	ω	Ω			

Hebrew Alphabet

Hebrew letters do not possess a capitalized form.

Name of letter	Ordinary	Script	Braille equivalent
aleph	א		
veth	ב		
gimel	ג		
daleth	ד		
heh	ה		
vav	ו		
zayin	ז		
cheth	ח		

Name of letter	Ordinary	Script	Braille equivalent
teth	ט		
yod	י		
chaph	כ		
lamed	ל		
mem	מ		
nun	נ		
samekh	ס		
ayin	ע		

Name of letter	Ordinary	Script	Braille equivalent
feh	פ		
tsadi	צ		
koph	ק		
resh	ר		
sin	ש		
thav	ת		

Russian Alphabet

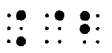
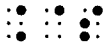
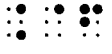
The Russian alphabet is sometimes referred to as the *Cyrillic alphabet*.

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	а	А	*а*	*А*	⠜
beh	б	Б	*б*	*Б*	⠃
veh	в	В	*в*	*В*	⠧
gheh	г	Г	*г*	*Г*	⠛
deh	д	Д	*д* or *э*	*Д*	⠙
yeh	е	Е	*е*	*Е*	⠑
zheh	ж	Ж	*ж*	*Ж*	⠚
zeh	з	З	*з* or *ƶ*	*З*	⠵
ee	и	И	*и*	*И*	⠊
kah	к	К	*к*	*К*	⠅
ell	л	Л	*л*	*Л*	⠇
em	м	М	*м*	*М*	⠍
en	н	Н	*н*	*Н*	⠝
oh	о	О	*о*	*О*	⠕
peh	п	П	*п*	*П*	⠏
err	р	Р	*р*	*Р*	⠗
ess	с	С	*с*	*С*	⠎
teh	т	Т	*т*	*Т*	⠞
oo	у	У	*у*	*У*	⠥
eff	ф	Ф	*ф*	*Ф*	⠋
khah	х	Х	*х*	*Х*	⠓
tseh	ц	Ц	*ц*	*Ц*	⠉
cheh	ч	Ч	*ч*	*Ч*	⠟
shah	ш	Ш	*ш*	*Ш*	⠱
shchah	щ	Щ	*щ*	*Щ*	⠭
yerih	ы	Ы	*ы*		⠮
eh	э	Э	*э*	*Э*	⠪
yu	ю	Ю	*ю*	*Ю*	⠳
yah	я	Я	*я*	*Я*	⠯

§23. Alphabets:

a. Specific provision is made in this Code for five alphabets — English, German, Greek, Hebrew, and Russian. The letters of the English alphabet are often called *Roman*, and those of the Russian alphabet *Cyrillic*.

b. Some of the letters of the ordinary lower-case Greek alphabet possess an alternative form. The more common ones are:

Name of letter	Sign	Braille equivalent
alpha	α	
beta	β	
theta	ϑ	
sigma	ς	
phi	φ	

When these alternative forms occur *instead of* the standard forms throughout a text, the symbols for the standard forms should be used in braille, and the transcriber should call attention to this usage by a transcriber's note. The alternative forms should be used only when the author has assigned distinct meanings to the standard and alternative forms of the same letter.

c. Some Greek letters used in textbooks are obsolete. The more common ones are:

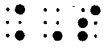
Name of letter	Sign	Braille equivalent
stigma	ς'	
vau	F	
koph (or qoph)	$\varphi\alpha\bigcirc$	
sampi	$\sampi$	

§24. Alphabetic Indicators:

a. Except for the English-letter indicator (see §§26-28), the appropriate alphabetic indicator must be used to specify the alphabet to which a letter belongs. If the letter is lower case, the corresponding alphabetic indicator must precede the letter directly; if the letter is capitalized, so that the capitalization indicator is also required, the alphabetic indicator must precede the capitalization indicator.

(1) α 

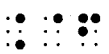
(Greek lower-case alpha)

(2) Σ 

(Greek capitalized sigma)

(3) π 

(Greek lower-case pi)

(4) φ 

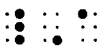
(alternative form of Greek lower-case phi)

(5) ς 

(Greek stigma; obsolete)

(6) $\mathfrak{a}$ 

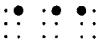
(German lower-case ah)

(7) $\mathfrak{A}$ 

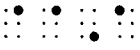
(German capitalized ah)

(8) $\aleph_0$ 

(Hebrew alef sub zero)

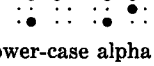
(9) a 

(Russian lower-case ah)

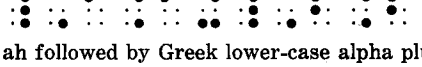
(10) A 

(Russian capitalized ah)

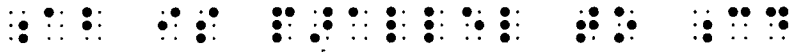
b. The effectiveness of an alphabetic indicator extends only to the letter or, in the case of the English-letter indicator, to a "short-form combination" or a lower-case Roman numeral which follows it. When an alphabetic indicator is required, it must be used with each individual letter of a sequence of letters or, in the case of the English-letter indicator, with the "short-form combination" which follows it. (See §25.)

(1) $\alpha\beta$ 

(Greek lower-case alpha followed by Greek lower-case beta)

(2) $\mathfrak{A}\alpha + \mathfrak{B}\beta$ 

(German capitalized ah followed by Greek lower-case alpha plus German capitalized beh followed by Greek lower-case beta)

(3) ab is parallel to cd 

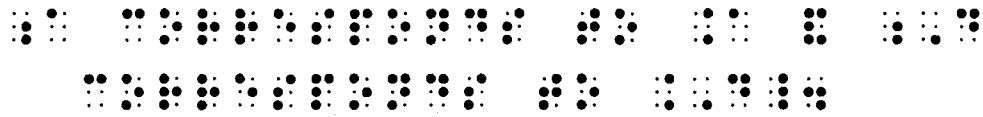
(4) iv 

§25. "Single Letters" and "Short-Form Letter Combinations" (See §26):

a. A "single letter", for the purposes of this Code, must meet the following requirements:

i. The letter must be from the English alphabet.

(1) a corresponds to α and D corresponds to Δ.



(a and D are "single letters"; α and Δ are not)

ii. It must be in regular type.

(1) **X** is a vector; x is a scalar

(**X** is not a "single letter"; x is a "single letter")

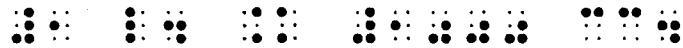
iii. It must be unmodified.

(1) x', x'', x_1, x_a, x^2, $\overline{x}$

(none of these items is a "single letter")

iv. It must not be an abbreviation or any of the words "a", "A", "I", or "O".

(1) 1 l. = 1000 cc.



(the l. is not a "single letter")

(2) From 11 a.m. to 2 p.m.

(the letters in a.m. and p.m. are not "single letters")

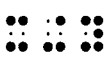
(3) If I take i as a variable . . .

(the words "I" and "a" are not "single letters")

v. It must be preceded by a space or by one or more punctuation marks in ink print. If this space is not shown in braille, the letter is no longer a "single letter." Whether these punctuation marks are preceded by a space or not is irrelevant. A grouping sign is not a mark of punctuation.

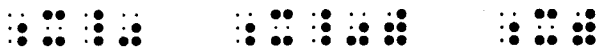
(1) x "x ("x (x

(of the above items, the first three are "single letters"; the last is not)

(2) x + y 

(y is not a "single letter" since it is not preceded by a space or punctuation)

vi. It must be followed by a space or by one or more punctuation marks in ink print. If this space is not shown in braille, the letter is no longer a "single letter." Whether these punctuation marks are followed by a space or not is irrelevant. A grouping sign is not a mark of punctuation.

(1) x" x") x) 

(of the above items, the first two are "single letters"; the last is not)

(2) A cos b or Acos b 

(A is not a "single letter" since no space will be left after the A in braille)

(3) x + y 

(x is not a "single letter" since it is not followed by a space or punctuation)

b. A "short-form letter combination", more briefly referred to as a "short-form combination", for the purposes of this Code must meet the following requirements:

i. It must be a letter combination which corresponds to a short-form word of English Braille.

ii. All of its letters must be lower case.

iii. It must meet the requirements of ii-vi of a above.

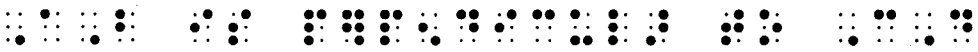
(1) cd is parallel to ef.

(cd is a "short-form combination"; ef is not)

(2) to the nth degree 

(nth is not a "short-form combination" according to i)

(3) AB is perpendicular to CD



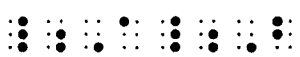
(neither AB nor CD are "short-form combinations" according to ii)

(4) 1 yr. = 12 mo. 

(yr. is not a "short-form combination" according to iv of a above)

§26. **Use of the English-Letter Indicator:** Each of the following rules applies, subject to the conditions of §27.

a. The English-letter indicator must be used with English letters, whether lower case or capitalized, if the type form is other than regular type.

(1) **AB** 

(boldface capitalized a followed by boldface capitalized b)

(2) *ab* 

(italic lower-case a followed by italic lower-case b)

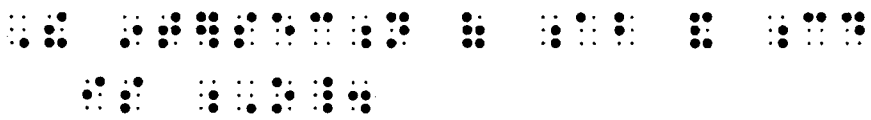
(3) *𝒆𝒇* 

(script lower-case e followed by script lower-case f)

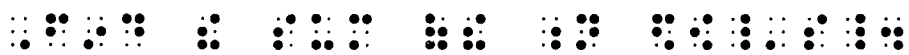
b. The English-letter indicator must be used with "single letters" or "short-form combinations" unless specifically prohibited by other rules of this Code.

(1) ∠'s A and B are acute.

(2) The intersection of ab and cd is O.



(3) Find the sum of the n ∠'s.



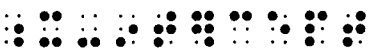
(here the shape sign for *angle* is not an omission sign so that the English-letter indicator is required)

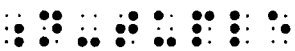
(4) (h ft, k ft) 

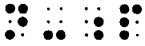
(this is not an "enclosed list"; therefore, the English-letter indicator is required with the h and the k)

(5) (a, 2x, y = z) 

(this is not an "enclosed list"; therefore, the English-letter indicator is required with the a but is not required with the y or z, as would be the case if the parentheses were not present)

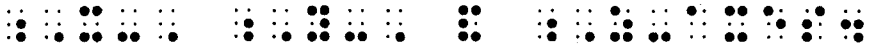
(6) x-intercept 

(7) n-tuple 

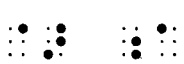
(8) not-p 

(9) Exercises A-F 

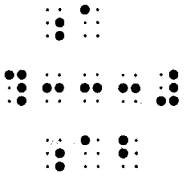
(10) Exercise 1-a 

(11) X-, Y-, and Z-axes. 

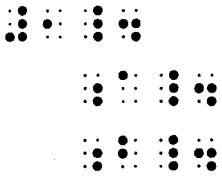
(12) ✓ a



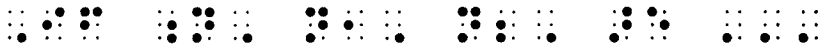
(13) $\frac{a}{ab}$



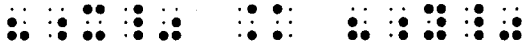
(14) 1.
a.
b.



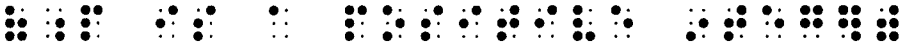
(15) If n, n_1, n_2, are . . .



(16) "x" = "y"



(17) (p is a positive integer)



(p is in direct contact with its left grouping sign; this letter would require the English-letter indicator if the parenthesis was removed)

(18) (p and q)

(p and q are in direct contact with their respective grouping signs; these letters would require the English-letter indicator if the parentheses were removed)

(19) (l, m, n are in set R)

(l is in direct contact with its left grouping sign and R is in direct contact with its right grouping sign; these letters would require the English-letter indicator if the parentheses were removed)

(20) (x-intercept)

(the x would require the English-letter indicator if the parenthesis was removed)

(21) (ab and cd)

(these "short-form combinations" would require English-letter indicators if the parentheses were removed)

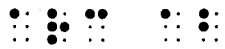
c. For use of the English-letter indicator with abbreviations, see §51.

d. For other situations in which the English-letter indicator may also be used, see §28.

§27. **Non-Use of the English-Letter Indicator:** It must not be assumed that because a symbol is a "single letter" or a "short-form combination" that the English-letter indicator must be used.

a. The English-letter indicator must not be used with a "single letter" or "short-form combination" that follows a function name or its abbreviation.

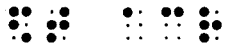
(1) cos A 

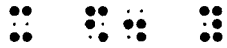
(2) arc ab 

(3) $e^{\sin x}$ 

b. The English-letter indicator must not be used with a "single letter" or "short-form combination" that follows a sign of shape, provided that the sign of shape does not have a plural or a possessive ending. The English-letter indicator must not be used with a "single letter" or "short-form combination" which precedes a sign of shape when that sign of shape is also a sign of omission.

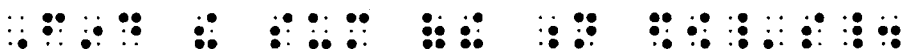
(1) ∠ a 

(2) △ acr 

(3) x □ y 

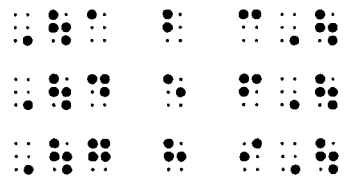
(the *square* is a sign of shape which is also a sign of omission)

(4) Find the sum of the n ∠'s.

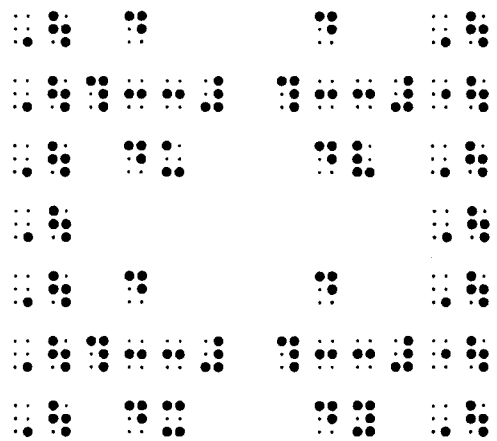


(here the shape sign for *angle* is not an omission sign so that the English-letter indicator is required)

c. The English-letter indicator must not be used before any letter or combination of two or more letters in a determinant or matrix.

(1) $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ 

(2) $\begin{vmatrix} \frac{d}{du} & \frac{d}{dv} \\ \frac{d}{dx} & \frac{d}{dy} \end{vmatrix}$

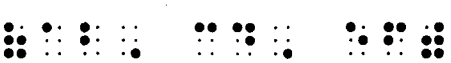


d. The English-letter indicator must not be used with a "single letter" or "short-form combination" which is an item in an "enclosed list." (For definition of "enclosed list" see §10.)

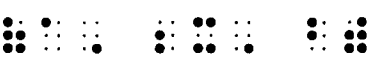
(1) (0, a, 1, b, 2)

(2) {a, b, c, d}

(3) (ab, cd, ef)

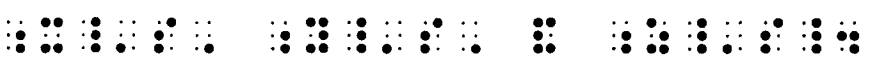


(4) (a, 2x, b)



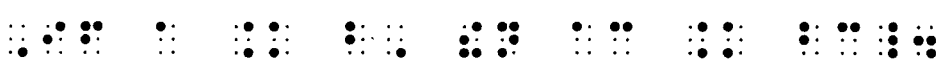
e. The English-letter indicator must not be used with the letter "s" when this "s" is part of the apostrophe-s combination.

(1) x's, y's, and z's.



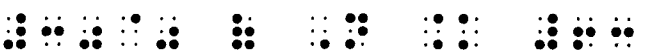
f. The English-letter indicator must not be used with a "single letter" or "short-form combination" which is preceded or followed by a comparison sign.

(1) If a = b, then ac = bc.



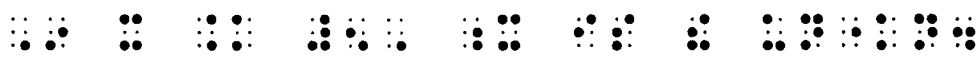
(2) a = b, but c $\neq$ b.

(3) 30% of N = 63



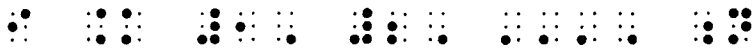
(4) "x = y"

(5) In x = 5, x is the unknown.



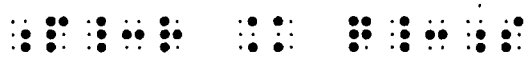
(6) For some value of s, d = st.

(7) a, b, and c = 10 

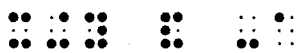
(8) i = 1, 2, . . . , n 

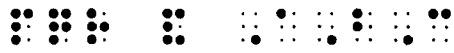
(9) n : v → r 

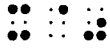
(10) e × e = e-squared 

(11) p:r = q:s 

g. The English-letter indicator must not be used with any letter or combination of letters which are neither "single letters" nor "short-form combinations" in situations not specifically covered (see §§26, 27, 28, and 51).

(1) x + y and —a 

(2) pqr and ABC 

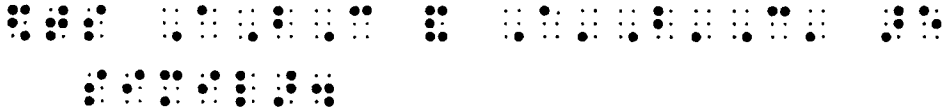
(3) x% 

(4) a cos B 

(5) m ∠ b 

(6) If n, n_1, n_2, are . . .

(7) ≙ ABC and A′B′C′ are similar.



h. For other situations in which the English-letter indicator is not required, see §28.

§28. **Other Considerations Concerning the English-Letter Indicator:**

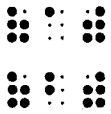
a. The English-letter indicator must not be used when only one letter or any combination of unspaced letters is in direct contact with both its opening and closing grouping signs, provided that they are English letters in regular type. When only one letter or any

combination of unspaced letters is in direct contact with only its opening or only its closing grouping sign, the English-letter indicator must be used (see §26) or must not be used (see §27) as though the grouping signs were not present. However, if the grouping sign has a prime, subscript or superscript, the English-letter indicator must not be used.

(1) 1.

(a)

(b)



(2) | x |, [x], ‖ f ‖

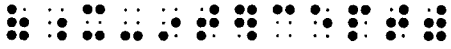
(3) (ab) + (cd)

(4) (p is a positive integer)

(5) (p and q)

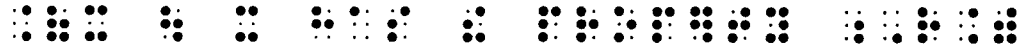
(6) (l, m, n are in set R)

(7) (x-intercept)



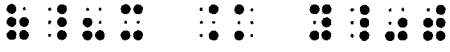
(8) (ab and cd)

(9) { x | x has the property R }



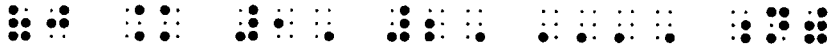
(because of the *such that* sign, which is a comparison sign, the x's do not require the English-letter indicator)

(10) ("x = y")

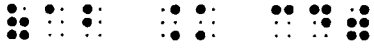


(11) Solve for x (x > y).

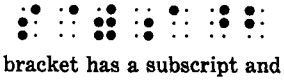
(12) (j = 1, 2, . . . , n)



(13) (ab = cd)

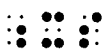


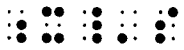
(14) $s]_a^b$

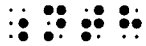


(closing bracket has a subscript and a superscript; therefore, the English-letter indicator is not required with the s)

b. When only one letter or any combination of unspaced letters has a plural, possessive or ordinal ending, the English-letter indicator must be used (see §26) or must not be used (see §27) as though such endings were not present.

(1) xs 

(2) x's 

(3) nth 

(4) 2nth 

(without the th ordinal ending the English-letter indicator would not be required before the n)

c. A lower-case Roman numeral must be treated as consisting of one letter even when it consists of more than one letter. The English-letter indicator must be used or must not be used in accordance with the rules for any letter (see §§26-28). A capitalized Roman numeral of one letter is subject to these same rules. For capitalized Roman numerals of more than one letter the English-letter indicator must not be used.

§29. Letters in Diagrams: When a single English letter in regular type is used as a label in a diagram, the English-letter indicator is required if the letter is in lower case, but must be omitted if the letter is capitalized.

§30. Letters in Tables: When letters appear in tables, whether as entries or headings, the English-letter indicator must be used or must not be used in accordance with the rules contained in §§26-28.

RULE V—TYPE FORMS

Type-Form Indicators for Letters, Numerals, and Compound Expressions

Boldface-Type 

Italic-Type 

Sanserif-Type 

Script-Type 

Type-Form Indicators for Words, Phrases, and Mathematical Statements

Opening Boldface-Type 

Opening Italic-Type 

Closing Boldface-Type 

Closing Italic-Type 

(For combinations of capitalization, alphabetic, and type-form indicators, see Appendix, page 208)

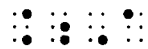
§31. Type Forms: Specific provision is made in this Code for five type forms — boldface, italic, regular, sanserif, and script. Except for regular type, these type forms must be specified by the appropriate type-form indicator.

§32. Use of Type-Form Indicators with Letters, Numerals, and Compound Expressions:

a. Subject to the provisions of §34, the appropriate type-form indicator must be used to express the type form of a letter. The type-form indicator for a letter must always be followed by an alphabetic indicator.

(1) *a* 

(italic English lower-case a)

(2) *A* 

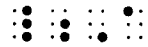
(italic English capitalized a)

(3) *a* 

(italic German lower-case ah)

(4) **a** 

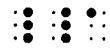
(boldface English lower-case a)

(5) **A** 

(boldface English capitalized a)

(6) **α** 

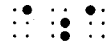
(boldface Greek lower-case alpha)

(7) **𝔞** 

(boldface German lower-case ah)

(8) **а** 

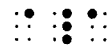
(boldface Russian lower-case ah)

(9) *𝒶* 

(script English lower-case a)

(10) *𝒜* 

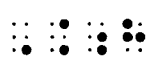
(script English capitalized a)

(11) *𝓪* 

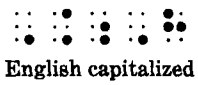
(script German lower-case ah)

(12) *𝓐* 

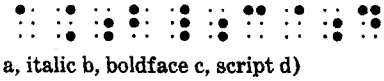
(script Hebrew alef)

(13) h 

(sanserif English lower-case h)

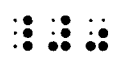
(14) H 

(sanserif English capitalized h)

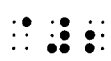
(15) *abcd* 

(regular a, italic b, boldface c, script d)

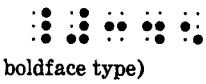
b. Subject to the provisions of §34, the appropriate type-form indicator must be used to express the type form of a numeral. The type-form indicator for a numeral must always be followed by the numeric indicator. If a numeral to be transcribed by using type-form indicators contains more than one digit, and is all of one type form, the type-form indicator and the numeric indicator must be used only before the first digit. If there is a transition from one type form to another non-regular type form within the same numeral, the new type-form indicator followed by the numeric indicator must be used before the first digit of the new type form. If the transition is to regular type, only the numeric indicator must be used.

(1) 0 

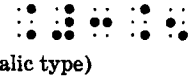
(boldface zero)

(2) *2* 

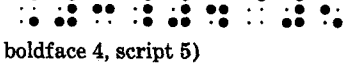
(script 2)

(3) 345 

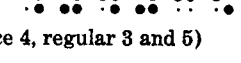
(345 in boldface type)

(4) *3.5* 

(3.5 in italic type)

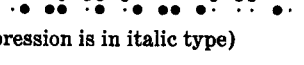
(5) *345* 

(italic 3, boldface 4, script 5)

(6) 435 

(boldface 4, regular 3 and 5)

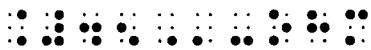
c. Subject to the provisions of §34, when a numeral is joined to a word or an abbreviation by a hyphen and the whole expression is printed in non-regular type, the appropriate type-form indicator must be used before the numeral only, but affects the entire compound expression. If there is a change in type form after the hyphen to regular type, the hyphen must be preceded by the literary termination symbol  (dots 6, 3). If there is a change in type form after the hyphen to non-regular type, only the appropriate type-form indicator must be used after the hyphen.

(1) *45-ohm* 

(the whole expression is in italic type)

(2) **45-ft** 

(the whole expression is in boldface type)

(3) *45*-ohm 

(45 in italic type, ohm in regular type)

(4) ***45*-ohm** 

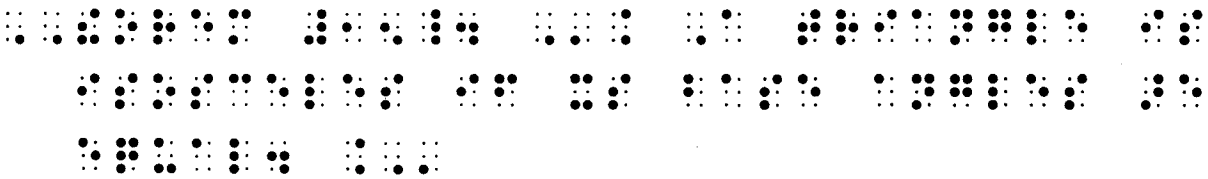
(45 in italic type, ohm in boldface type)

d. When a type-form indicator is used with letters, its effectiveness extends only to the letter which follows it. Thus, except for regular type, a type-form indicator must be used with each individual letter of a sequence of letters. When a type-form indicator is used with numerals only, it is effective until terminated by a space, a numeric indicator, or any non-numeric symbol. When a type-form indicator is used with a compound expression, it is effective for the entire compound expression unless terminated by the literary termination symbol  (dots 6, 3), or another type-form indicator.

§33. Use of Type-Form Indicators with Words, Phrases, and Mathematical Statements:

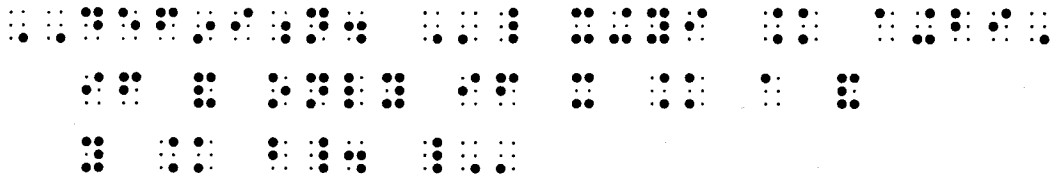
a. When the ink-print text uses the convention of showing *labeled statements* such as theorems, definitions, axioms, lemmas, etc. in non-regular type form, the body of such an item must be transcribed using the corresponding type-form indicators, but the labels themselves must be transcribed as though they were entirely capitalized. If, in the body of the labeled statement, a word or phrase is singled out for special attention by using a non-regular type form for the purpose of definition or other elaboration, such a statement must also be transcribed using the corresponding type-form indicators. When the passage to be transcribed is entirely of the same non-regular type form, it must be preceded by the appropriate opening type-form indicator and followed by the corresponding closing type-form indicator. These type-form indicators must be separated from the enclosed material by one space. If material in non-regular type other than letters or formulas constituting a mathematical expression is embedded within a larger body of a different non-regular type, the embedded material must be transcribed using the appropriate type-form indicators in accordance with the procedure described above. If it becomes necessary to use two of these type-form indicators consecutively, they must be unspaced from each other.

(1) **Theorem 15.** *A triangle is isosceles if its base angles are equal.*



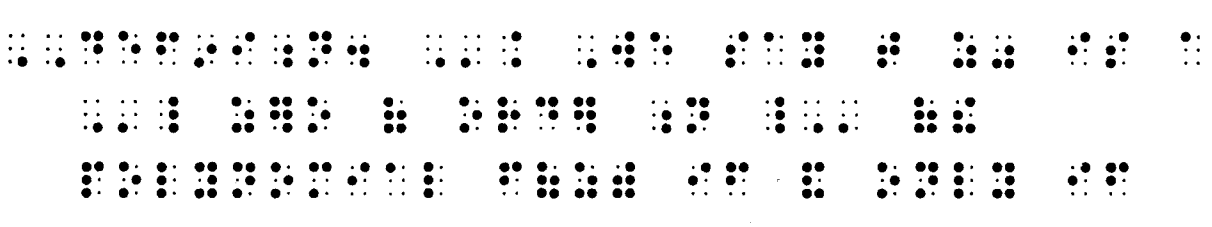
(in ink print, "Theorem 15" is in boldface type and only the first letter of "Theorem" is capitalized)

(2) **Definition. $x + yi = a + bi$, if and only if $x = a$ and $y = b$.**



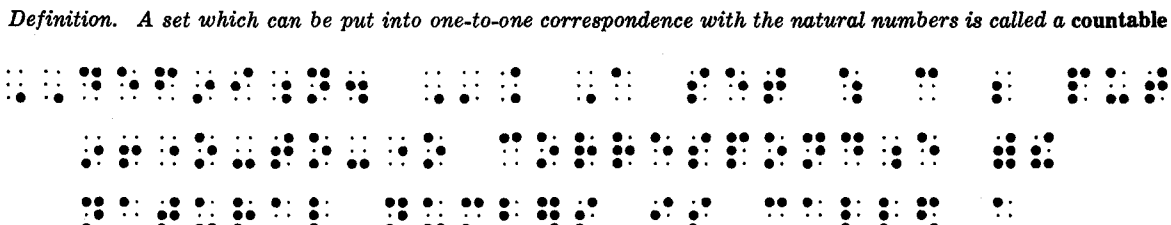
(in ink print, "Definition" is in boldface type and only its first letter is capitalized)

(3) **Definition.** *We say that z_0 is a* **zero of order n** *of the polynomial f(z) if and only if . . .*



(in ink print, "Definition" is in boldface type and only its first letter is capitalized)

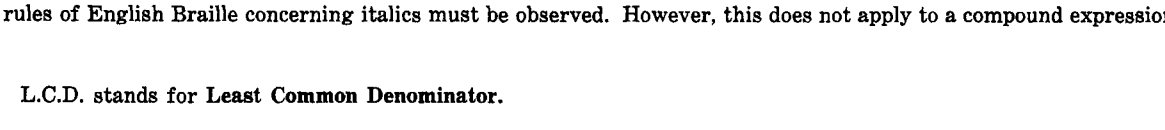
(4) *Definition. A set which can be put into one-to-one correspondence with the natural numbers is called a* **countable set.**



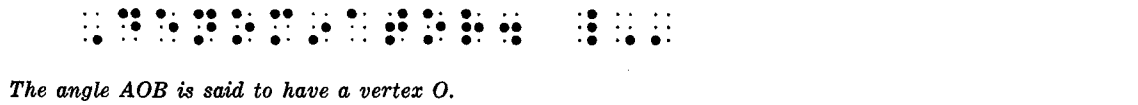
(in ink print, "Definition" is italicized and only its first letter is capitalized)

b. Subject to the provisions of §34, when the ink-print text shows a boldface word or phrase within an unlabeled item, or when it shows an italicized phrase which either begins or ends with a mathematical expression, the appropriate type-form indicators must be used according to the rules in a above. When the ink print shows an italicized word or an italicized phrase which both begins and ends with a word, the rules of English Braille concerning italics must be observed. However, this does not apply to a compound expression (see §32c).

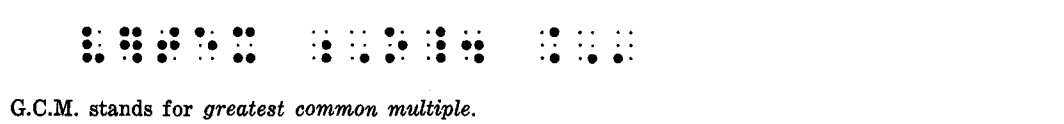
(1) L.C.D. stands for **Least Common Denominator.**



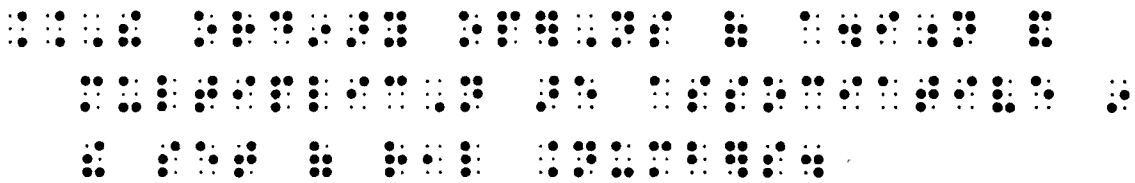
(2) *The angle AOB is said to have a vertex O.*



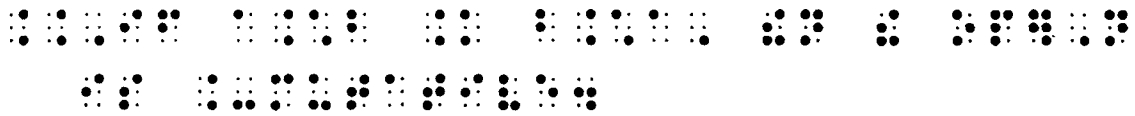
(3) G.C.M. stands for *greatest common multiple.*



(4) *The ordinary operations of addition and multiplication are associative in the set of real numbers.*



(5) *If* $a \circ b = b \circ a$, *then the operation is commutative.*



§34. Non-Use of Type-Form Indicators:

a. A type-form indicator must not be used when a letter or a numeral is printed in regular type.

b. When any material, mathematical or literary, is printed in non-regular type that has no mathematical significance, the variant type form must not be represented in the transcription. Frequently, it is the practice to print the letters of all formulas throughout a book in italicized type. This practice must not be carried over to the transcription unless the author has specifically distinguished between two meanings of the same letter, assigning one meaning to the letter in regular type and another to the letter in italic type. In addition, a variant type form is often used, particularly at the lower grade levels, for the sole purpose of attracting the reader's attention. Such variant type forms must also not be represented in the transcription.

§35. Boldface Type:

a. When certain signs of operation or comparison are printed in boldface type, this Code employs the device of placing dots 4-5-6 before the corresponding symbol. The specific signs to which this technique applies are listed in appropriate sections throughout the Code, and the transcriber must not use this technique with any other sign. When used in this way, dots 4-5-6 must not be regarded as the boldface type-form indicator but as an integral part of the symbol to which it belongs. This technique has been used only when the distinction between the regular and boldface forms of the same sign has mathematical significance. Dots 4-5-6 are also used as part of the technique for representing filled-in shapes (see §108).

b. Boldface type, used in many texts to identify letters as vectors, must be preserved in the transcription. When both boldface type and arrows of uniform construction are used in conjunction to represent vectors, the arrows themselves must be omitted from the transcription unless the author calls special attention to them as a notational device, but a transcriber's note must be included indicating their presence in the ink print copy.

RULE VI—PUNCTUATION SIGNS AND SYMBOLS

Punctuation Indicator 

Punctuation Marks

Apostrophe ' 

Colon : 

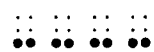
Comma

Literary , 

Mathematical , 

Dash

Short — 

Long —— 

Ellipsis ... 

Exclamation Point ! 

Hyphen - 

Period . 

Question Mark ? 

Quotation Marks

Left inner ‘ 

Left outer “ 

Right inner ’ 

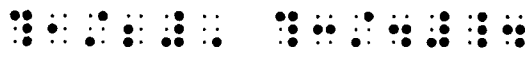
Right outer ” 

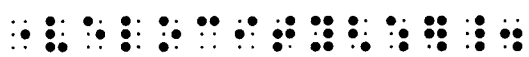
Semicolon ; 

§36. Modes of Punctuation: Since numerals are represented by symbols in the lower part of the cell, and since these symbols also serve as punctuation marks, it is necessary to formulate rules concerning punctuation so that the meanings of such symbols are unambiguous. This Code employs two modes of punctuation — mathematical and literary.

§37. Use of the Punctuation Indicator: Subject to the provisions of §38, the punctuation indicator must be used *before* a punctuation mark and *after* any symbol of the type listed below. In all these circumstances, the mode of punctuation is considered to be mathematical.

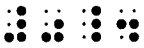
i. After any braille indicator.

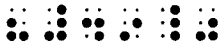
(1) $\frac{1}{2}$, $\frac{3}{4}$. 

(2) $\overline{\text{velocity}}$. 

(bar over "velocity")

ii. After any numeric symbol written as in the Nemeth Code.

(1) 0. 

(2) "49" 

iii. After a Roman numeral.

(1) I, II, III. 

iv. After a dash or ellipsis, when these occur in a mathematical context. When the nature of the context is in doubt, the punctuation indicator must be used.

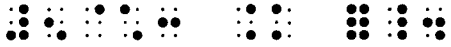
(1) $24 = 6 + \text{——}$. 

(2) 1, 3, ... 

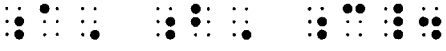
v. After any reference symbol.

(1) note*. 

vi. After the general omission symbol.

(1) $5 \times 3 = ?$. 

vii. After a "single letter."

(1) a, b, c. 

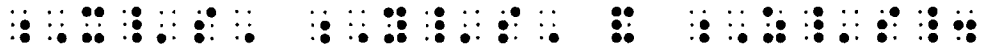
viii. After a sequence of more than one letter in which each letter has a separate identity, provided that such a sequence is not an abbreviation.

(1) △ ABC. 

ix. After ordinal, plural, or possessive endings which are joined to numerals, letters, or other mathematical expressions.

(1) 1st, 2nd, 3rd, 4th.

(2) X's, Y's, and Z's.

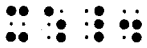


x. After any word or abbreviation which is not on the base line, if the punctuation which follows is on the base line.

(1) 18_{seven}.

xi. After any modified expression.

(1) $\overline{x}$.

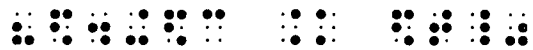


xii. After the radical symbol.

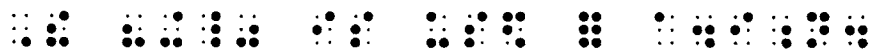
(1) "$\sqrt{\ }$" means "square root."

xiii. After any symbol of shape or shape modification, operation, or comparison.

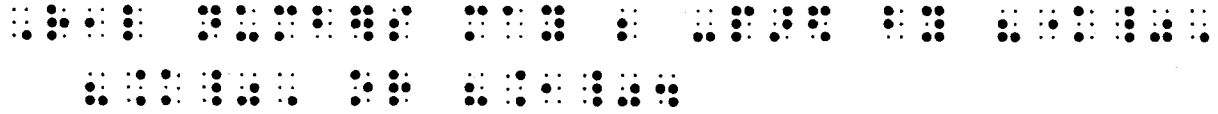
(1) "$\square + \bigcirc = \triangle$"



(2) The "+" is used for addition.



(3) Real numbers may be compared by "<", "=", or ">".



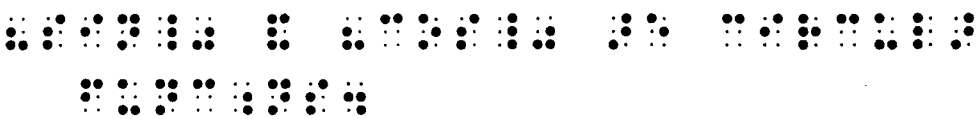
xiv. After any symbol of grouping whether brailled or drawn in.

(1) ("8").

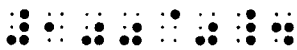
(2) (LCD).

xv. After any abbreviated function name or unabbreviated function name, provided that the latter occurs in a mathematical context.

(1) "sin" and "cos" are circular functions.

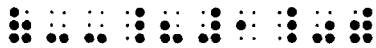


xvi. After any of the miscellaneous symbols of Rule XXII.

(1) 100%. 

xvii. After a comma, hyphen, or dash, provided that if these were removed and the space which they occupy were not present, one of the conditions i-xvi would apply.

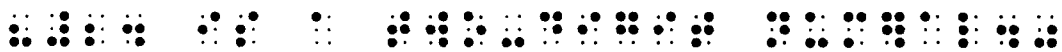
(1) 0," 

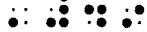
(2) (—"1") 

§38. **Non-Use of the Punctuation Indicator:** It must not be assumed that because a punctuation mark occurs that the punctuation indicator must be used. The punctuation indicator must not be used under any of the circumstances listed below. In all these circumstances, the mode of punctuation is considered to be literary.

i. At the beginning of a braille line or after a space.

(1) "24 is a two-digit numeral."



(2) '49 

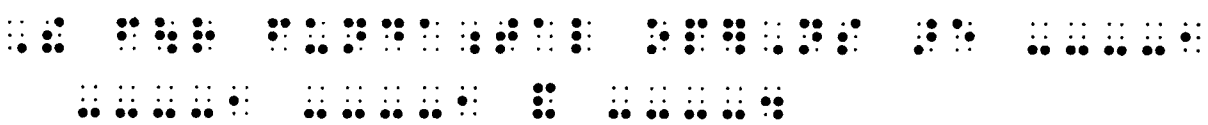
ii. After any numeric symbol written as in English Braille.

(1) Copyright 1970. 

(item on a title page)

iii. After a dash or ellipsis, when these occur in a literary context.

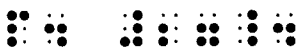
(1) The four fundamental operations are ——, ——, ——, and ——.

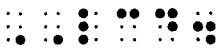


(2) five and three are . . . . 

iv. After a word or abbreviation provided that the punctuation is at the same level as that word or abbreviation.

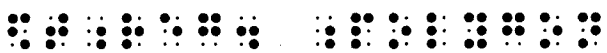
(1) e.g. 

(2) p. 27. 

(3) LCD. 

(4) mi./min. 

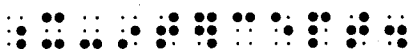
(5) {Wed., Thurs., Fri.}

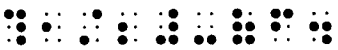
(6) $\triangle_{\text{reg. polygon}}$ 

(7) 2 quarts. 

(8) ("three") 

(9) 5-cent. 

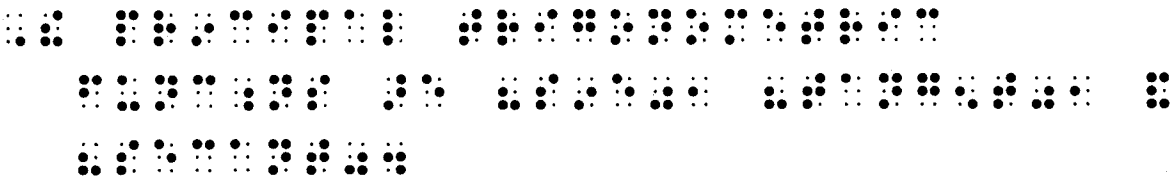
(10) x-intercept. 

(11) ½-off. 

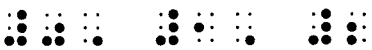
(12) rate × time. 

v. After any unabbreviated function name which occurs in a literary context.

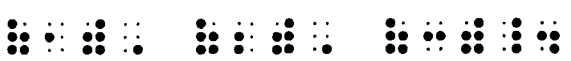
(1) The principal trigonometric functions are "sine", "tangent", and "secant".



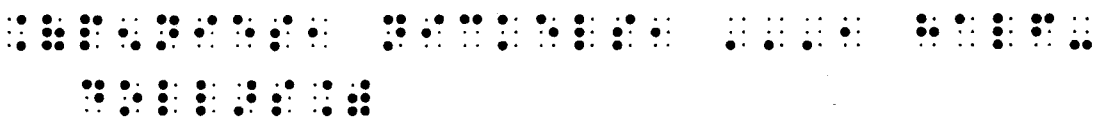
vi. Before a comma, hyphen, dash, or ellipsis.

(1) 0, 1, 2 

(2) 1's, 2's, and 3's.

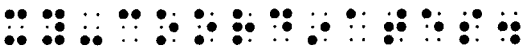
(3) (1), (2), (3). 

(4) {pennies, nickels, . . . , half-dollars}



(5) Transcribed, 1970, by

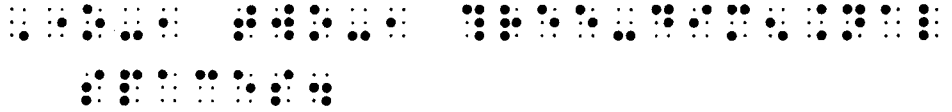
(item on a title page)

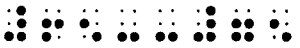
(6) xy-coordinates. 

(7) Exercises 30-40. 

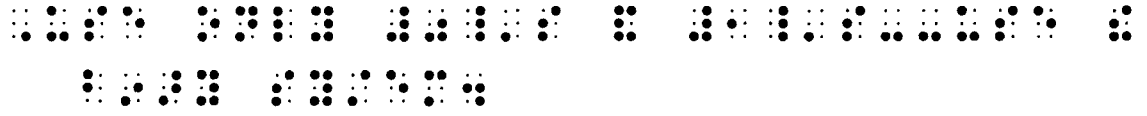
(8) 1-, 2-, and 3-dimensional spaces.

(9) One-, two-, three-dimensional spaces.

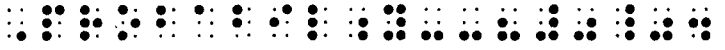


(10) 65—75 

(11) Use only 0's and 1's—use the binary system.

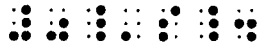


vii. Before any except the first punctuation mark in a sequence of punctuation marks which requires the use of the punctuation indicator.

(1) Probability—"0". 

(2) 0."

§39. Plural and Possessive Endings: The apostrophe-s combination may be joined to numerals, letters, and other mathematical expressions to form their plurals or possessives. When, in ink print, the apostrophe has been omitted, it likewise must be omitted in the transcription. The choice between the singular and plural form of a word is sometimes shown by enclosing an "s" within parentheses.

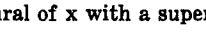
(1) 0's. 

(2) A's, B's, and C's.

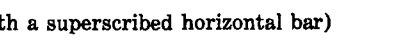
(3) 1s, 2s, and 3s. 

(4) $\widetilde{x}$'s 

(the plural of x with a superscribed tilde)

(5) $\bar{x}$'s 

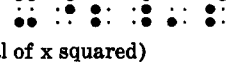
(the plural of x with a superscribed horizontal bar)

(6) $\angle$s 1 and 2 

(7) $\triangle$ ABC and DEF 

(8) x^2's 

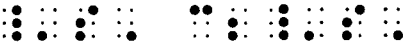
(the plural of x squared)

(9) c_i's 

(the plural of c sub i)

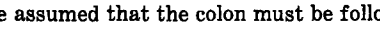
(10) The c_1's, c_2's, . . . , c_n's.

(the plurals of c sub 1, c sub 2, . . . , c sub n)

(11) principle(s) 

§40. **Colon:** It must not be assumed that the colon must be followed by a space as is generally the case in English Braille.

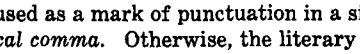
(1) 3:30

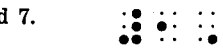
(2) f:(x, y) 

§41. **Comma:**

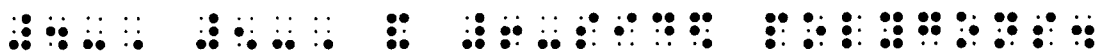
a. When a comma is used as a mark of punctuation in a situation in which the mode of punctuation is mathematical, the comma is referred to as the *mathematical comma.* Otherwise, the literary comma must be used.

(1) 1, 3, 5, and 7.

(2) "x, y" 

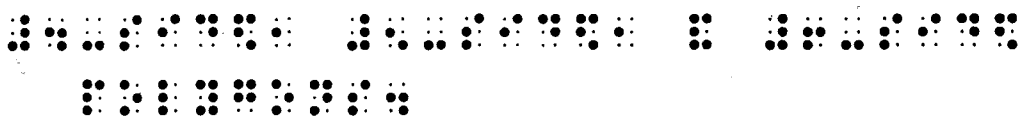
(3) "3", 

(4) 4-, 5-, and 6-sided polygons.

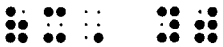


(no space after comma in ink print; in braille, space required after a comma used as a punctuation mark)

(5) 4-sided, 5-sided, and 6-sided polygons.



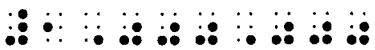
(6) i.e., 

(7) (x,y) 

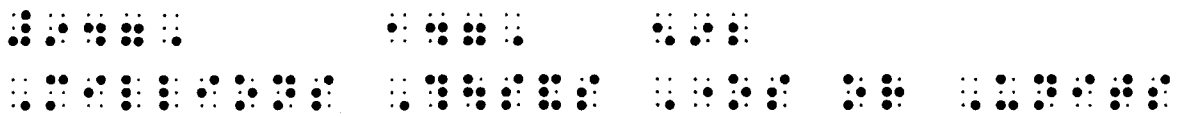
(8) (-3,2) 

(no space after comma in ink print; in braille, space required after a comma used as a punctuation mark)

b. No space must be left after the comma which is used as a numeric symbol except for the purpose of achieving alignment.

(1) 1,000,000 

(2) 947, Millions 147, Thousands 592 Ones or Units



§42. **Dash (Long):** The long dash must be preceded and followed by a space. However, no space may be left between the long dash and any of the items listed below, provided these items apply to the long dash.

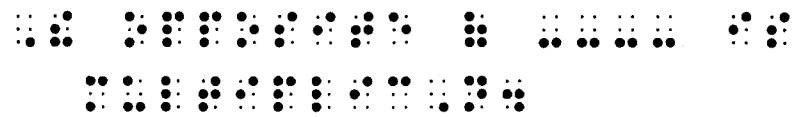
i. Symbols of punctuation other than the hyphen.

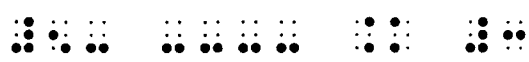
ii. Braille indicators.

iii. Symbols of grouping.

iv. The symbols for decimal, dollars, cents, percent, pounds (sterling), and primes.

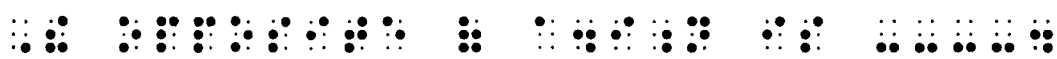
(1) The opposite of —— is multiplication.



(2) $5 - \text{——} = 3$ 

(the minus is not one of the listed items)

(3) The opposite of addition is ——.



(4) $\frac{\text{——}}{15} = \frac{2}{3}$ 

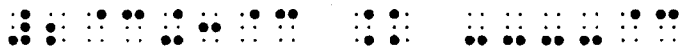
(the opening fraction indicator applies to the dash; the fraction line is not one of the listed items)

(5) (——, 4, 6, 8, ——) 

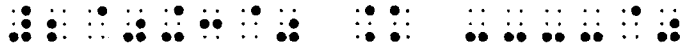
(the opening and closing grouping symbols apply to their respective dashes)

(6) $\$2 + \$3 = \$\text{——}$ 

(the dollar sign applies to the dash)

(7) $2¢ + 3¢ = \text{——}¢$ 

(the cent sign applies to the dash)

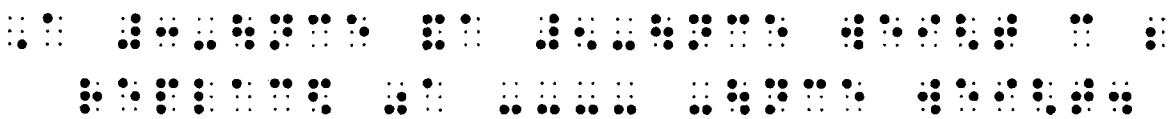
(8) $2\% + 3\% = \text{——}\%$ 

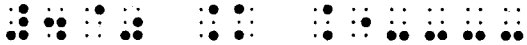
(the percent sign applies to the dash)

(9) £2 + £3 = £—— 

(the pound (sterling) sign applies to the dash)

(10) A 3-ounce and a 5-ounce weight can be replaced by a —— -ounce weight.



(11) 4% = .—— 

(the decimal point applies to the dash)

(12) 12′ = ——″ 

(the double prime applies to the dash)

§43. Ellipsis:

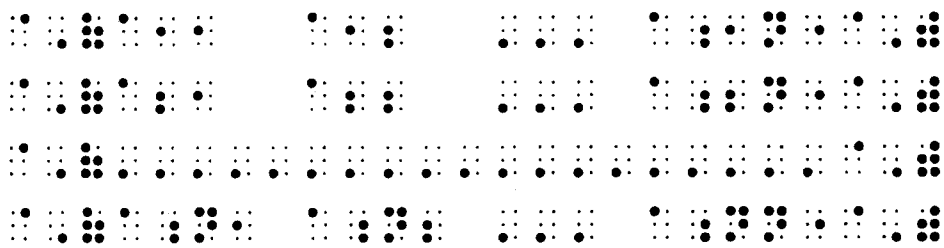
a. Any dot or series of dots in print which represents an omitted term, entry, or line is an ellipsis. It must be represented in braille by a minimum of three dots.

(1) 1, 3, 5, . . . , 15. 

(2) Mary, Sally, ...

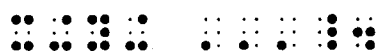
(3) a, ar, ar^2, ...

(4) $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$



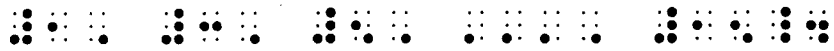
b. The ellipsis is subject to the same spacing rules as the long dash. See §42.

(1) $x + y + \dots$



(the period applies to the ellipsis)

(2) 1, 3, 5, . . . , 15.

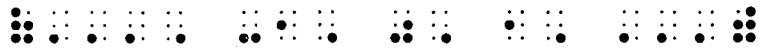


(the comma applies to the ellipsis)

(3) $p_1^{\alpha_1} \dots p_r^{\alpha_r}$

(the base-line indicator applies to the ellipsis)

(4) (. . . , -1, 0, 1, . . .)



(the opening and closing grouping symbols apply to their respective ellipses)

(5) $12¢ + 14¢ = \dots ¢$

(the cent sign applies to the ellipsis)

§44. **Exclamation Point:** The exclamation point is represented by the same sign of ink print as the factorial sign. The context is usually sufficiently clear in regard to this distinction so that the possibility of doubt in choosing the proper symbol is small.

§45. **Hyphen:** The hyphen is represented by the same sign of ink print as the minus sign. Since the corresponding braille symbols also coincide, a minimum of decision-making in this regard is required of the transcriber. A space must be left between a hyphen and an adjacent dash.

RULE VII—REFERENCE SIGNS AND SYMBOLS

General Reference Indicator

Asterisk *

Dagger

Single †

Double ‡

Paragraph Mark ¶

Section Mark

Single §

Double §§

Star ☆

§46. Reference Signs and Symbols: The reference signs of this section must be represented by the symbols listed above and English Braille symbols must not be used. Some of these signs are also used as signs of operation and in that case the rules governing signs of operation apply (see **Rule XIX**). When it is certain that a symbol in the above list is to be used for reference purposes, the superscript position, if indicated in ink print, must be ignored in the transcription.

When a reference sign occurs for which no provision exists in this Code, such as darts, pictures, etc., the transcriber must devise a suitable symbol with an explanatory transcriber's note. Whether a reference symbol exists in this Code or has been devised by the transcriber, such symbols are subject to the rules for signs and symbols of reference.

(1) A Cantor* set is . . .

(the asterisk is a reference sign, and its superscript position must not be indicated)

(2) f*g

(the asterisk is a sign of operation)

§47. General Reference Indicator: When reference to a footnote is denoted by a numeral, usually in the superscript position, and no other reference sign is employed, the general reference indicator immediately followed by the numeral of the printed text must be used in the transcription and the numeral must not be represented as being in the superscript position.

(1) Find the index[1] of the radical.

(in ink print, a 1 appears in the superscript position after "index"; it refers to a footnote)

§48. Spacing with Symbols of Reference:

a. When a reference sign which calls attention to or introduces a footnote is attached to a word or mathematical expression, the reference symbol must follow that word or expression with a space between. If such a reference sign is unattached its position relative to its surrounding material must be preserved, and a space must be left on either side of the reference symbol. However, if there is punctuation which applies to such a reference, no space should be left between the reference symbol and the punctuation mark which applies to it.

(1) *Irrational numbers . . .

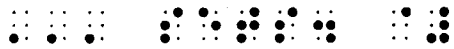
(in ink print, the asterisk precedes "irrational" and is unspaced from it)

(2) Irrational* numbers . . .

(in ink print, the asterisk follows "irrational" and is unspaced from it)

(3) * Irrational numbers . . .

(in ink print, the asterisk precedes "irrational" and is spaced away from it)

(4) . . . sets.* 

(in ink print, the asterisk follows the period but the period does not apply to the asterisk)

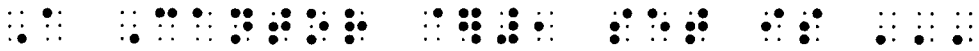
(5) . . . sets*. 

(in ink print, the asterisk follows "sets" and is unspaced from it; the period applies to the asterisk)

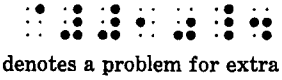
b. The symbols for the section mark, paragraph mark, general reference indicator, star, asterisk, and dagger, when the asterisk or dagger does not call attention to or introduce a footnote, must be unspaced from the letter or numeral which applies to them. All reference symbols, whether or not they call attention to or introduce a footnote, must be spaced away from the words to which they apply.

(1) §1, ¶a 

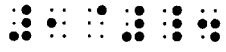
(2) A Cantor[1] set is . . .



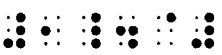
(in ink print, a 1 appears in the superscript position after "Cantor"; it refers to a footnote)

(3) *10. 

(asterisk denotes a problem for extra study and is followed by the problem number)

(4) 1*. 

(asterisk denotes a problem for extra study and follows the problem number)

(5) 1.* 

(asterisk denotes a problem for extra study and follows the period; the numeral 1 applies to the asterisk even though there is an intervening period)

(6) * For extra credit. 

(this is a footnote)

c. The placement and indentation of footnotes are subject to the rules for footnotes in the "Code of Braille Textbook Formats and Techniques."

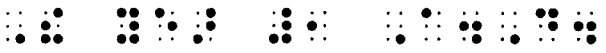
RULE VIII—ABBREVIATIONS

§49. Abbreviations:

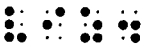
a. Abbreviations must be regarded in a broad sense to include the following items:

i. Universal literary abbreviations of the type commonly listed in a dictionary.

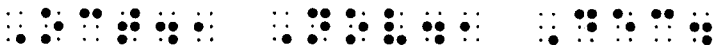
(1) 11 A.M. 

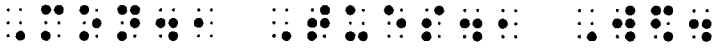
(2) The year 1 A.D. 

(3) 1/x vs. T 

(4) viz. 

(5) Ph.D. 

(6) Oct., Nov., Dec. 

(7) Mon., Tues., Wed. 

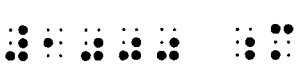
(8) 110 W. 110th St. 

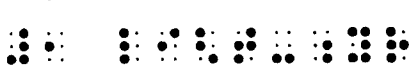
ii. Abbreviations of measurement.

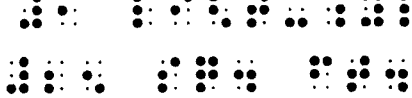
(1) 1 yd. 

(2) C. stands for Centigrade 

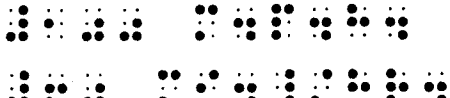
(3) 980 g. 

(4) 1000 m 

(5) 1 light-yr 

(6) 25 sq. ft. 

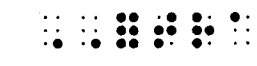
(7) 100 m.p.h. 

(8) 60 mi./hr. 

(9) 6 ft.-lbs. 

iii. Acronyms.

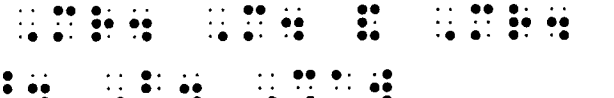
(1) FORTRAN

(2) ASCAP 

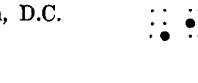
iv. Personal or geographic initials.

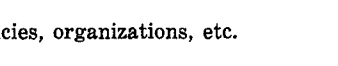
(1) I saw Mr. M. and Mr. N.

(2) G. B. Shaw 

(3) Washington, D.C. 

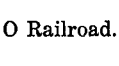
v. Initials of agencies, organizations, etc.

(1) RCA 

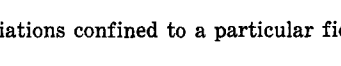
(2) B.V.D. 

(3) The B & O Railroad.

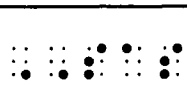
vi. Special abbreviations confined to a particular field or even to a particular book.

(1) lcd 

(means "least common denominator")

(2) L.U.B. 

(means "least upper bound")

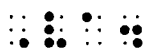
(3) SAS 

(means "side-angle-side")

(4) ClPA 

(means "Closure Property for Addition")

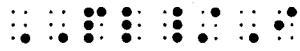
vii. Abbreviations formed by the use of initial or principal letters of a word, phrase, or name.

(1) Va. 

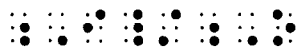
(means "Virginia")

(2) n st. angles 

("st." means "straight")

(3) PL/I 

(means "Program Language I")

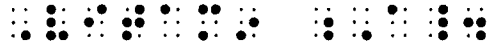
(4) I/O 

(means "Input-Output")

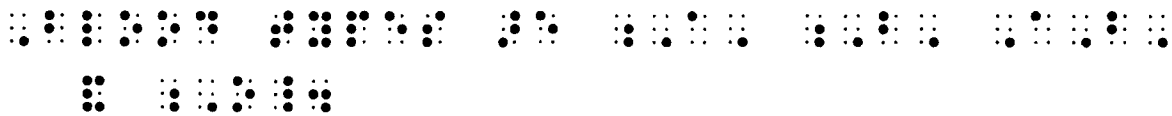
(5) d-c 

(means "direct current")

b. When a letter or sequence of letters does not represent a word or phrase, it must not be considered as an abbreviation and must be transcribed according to other rules of this Code. Abbreviated function names, as well as model numbers, serial numbers, etc. must also not be considered abbreviations and must be transcribed according to other rules of this Code. When there is doubt as to whether or not a construction is an abbreviation, it must be treated as if it were not an abbreviation.

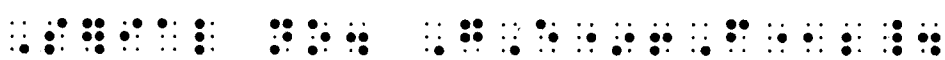
(1) Vitamin A. 

(2) Blood types are A, B, AB, and O.

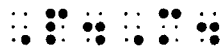


(3) sin x 

(4) Serial no. GE96F12.



§50. Capitalization with Abbreviations: In an abbreviation, whenever letters are capitalized in ink print, a single letter must be preceded by the single capitalization indicator, and a sequence of more than one letter must be preceded by the double capitalization indicator.

(1) P.M. 

(2) EST 

(means "Eastern Standard Time")

§51. English-Letter Indicator with Abbreviations:

a. When a period follows an abbreviation, there are four possibilities to consider:

i. The period applies to the abbreviation but does not end a sentence.

ii. The period ends a sentence but does not apply to the abbreviation.

iii. The period both applies to the abbreviation and ends a sentence.

iv. It is doubtful whether the period applies to the abbreviation.

In the case of ii, the English-letter indicator must be used or must not be used as if the period were not present. In case iv, the period should be considered as applying to the abbreviation and the appropriate rule must then be applied.

The use or non-use of the English-letter indicator with abbreviations does not depend upon the braille symbols with which the abbreviation may happen to be in contact, such as grouping symbols, braille indicators, fraction lines, the hyphen, or the slash.

b. The English-letter indicator must be used before an abbreviation which consists of one letter or of a combination of letters corresponding to a short-form word provided the abbreviation is not followed by a period which applies to it.

(1) 10 g + 10 g = 20 g

(no periods apply to these abbreviations)

(2) We know 32°F = 0°C.

(the period ends a sentence and does not apply to the abbreviation)

(3) 1 light-yr 

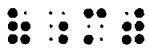
(no period applies to "yr" whose letters correspond to a short-form word)

(4) lat. 30°20′ N 

(no period applies to the "N")

(5) □l = 1000 cc 

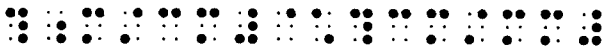
(no period applies to the "l")

(6) (m) 

(the presence of the parentheses has no effect upon the decision that the English-letter indicator must be used)

(7) □$m^2 = 100\ cm^2$ 

(no period applies to the "m"; the presence of the superscript indicator has no effect upon the decision that the English-letter indicator must be used)

(8) $\frac{m}{cm} \times \frac{cm}{mm}$ 

(no period applies to the "m"; the presence of the fraction indicator and fraction line has no effect upon the decision that the English-letter indicator must be used)

(9) I/O means "Input/Output."

(no period applies to either abbreviation; the presence of the slash has no effect upon the decision that the English-letter indicator must be used)

c. The English-letter indicator must not be used before an abbreviation which consists of one letter or of a combination of letters corresponding to a short-form word provided the abbreviation is followed by a period which applies to it. The English-letter indicator must also not be used before an abbreviation whose letters do not correspond to a short-form word. In this case, whether a period applies to the abbreviation or not has no effect on the rule for the non-use of the English-letter indicator.

(1) 100° C. = 212° F.

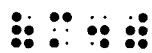
(the periods apply to the abbreviations)

(2) Does 1 km. = 1000 m.?

(the period applies to the "m")

(3) 1 light-yr. 

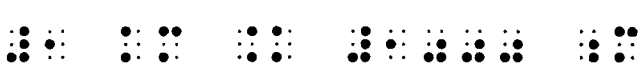
(the period applies to "yr" whose letters correspond to a short-form word)

(4) (m.) 

(the presence of the parentheses has no effect upon the decision that the English-letter indicator must not be used)

(5) $\frac{w.}{v.}$ 

(the periods apply to the abbreviations; the presence of the fraction indicators and fraction line has no effect upon the decision that the English-letter indicator must not be used)

(6) 1 km = 1000 m 

(the abbreviation "km" does not correspond to a short-form word)

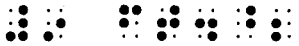
(7) 100 cm.^{2} = ? m.^{2} 

(the periods apply to the abbreviations)

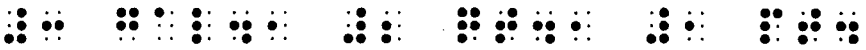
§52. **Punctuation with Abbreviations:** Abbreviations must be punctuated in the literary mode, provided that the punctuation is at the same level as that abbreviation. (See §§37 and 38.)

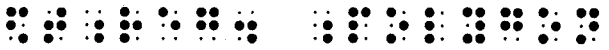
(1) i.e., 

(2) (e.g., . . .) 

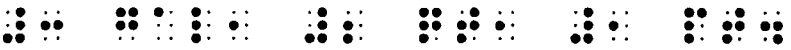
(3) 9 ft.^{2} 

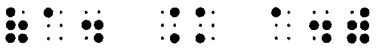
(4) 3 gal., 2 qt., 1 pt.



(5) $\triangle_{\text{reg. polygon}}$ 

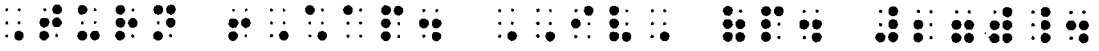
(6) 3 gal, 2 qt, 1 pt.



(7) (a. = a.) 

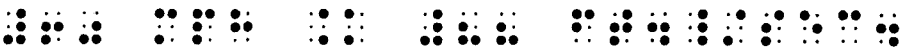
(abbreviated form for angle = angle)

(8) Turn to Chap. IV, (p. 27).



(9) Refer to Vol. I, pp. 30-35.

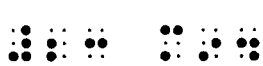
(10) 60 mph = 88 ft./sec.

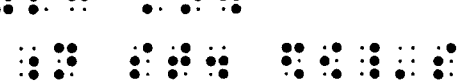


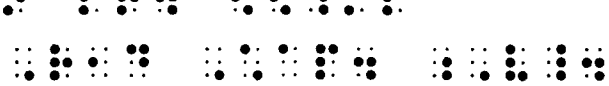
§53. **Contractions in Abbreviations:** No contractions may be used in an abbreviation which is in direct contact with any of the items in §55a. The abbreviation "in." or "in" usually meaning "inches" must never be contracted. The "st" contraction may only be used for abbreviating "street" or "saint". It must not be used for any other abbreviation, such as "st." for "straight".

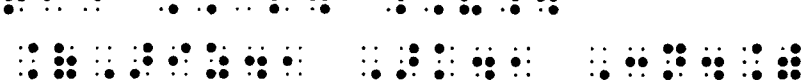
(1) 15 in. 

(2) 15-in 

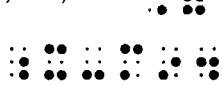
(3) 23 min. 

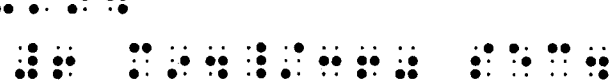
(4) n st. ∠'s 

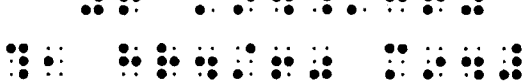
(5) Read Chap. V. 

(6) {Ariz., Ark., Conn.} 

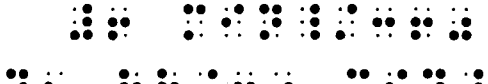
(7) {Wed., Thurs., Fri.} 

(8) x-min. 

(9) 6 min./360 sec. 

(10) $\frac{1\text{ hr.}}{60\text{ min.}}$ 

(11) 6 min/360 sec 

(12) $\frac{1\text{ hr}}{60\text{ min}}$ 

(13) $S_{\text{part. sum}}$ 

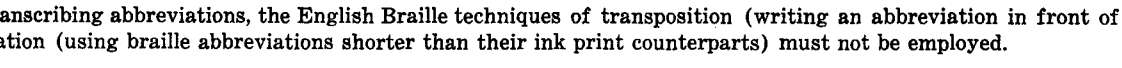
(period at the subscript level)

(14) statvolt-cm/statamp-oersted

§54. Spacing with Abbreviations:

a. In transcribing abbreviations, the English Braille techniques of transposition (writing an abbreviation in front of its number) and condensation (using braille abbreviations shorter than their ink print counterparts) must not be employed.

(1) Turn to Chap. IV, (p. 27).

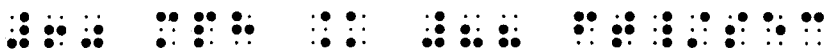


(2) Refer to Vol. II, pp. 30-35.

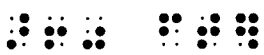
b. No space should be left between an abbreviation and its period, if present, and a slash line or any symbol of grouping, indicator, punctuation, or fraction line which applies to the abbreviation. A space must also not be left between two compo-

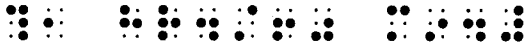
nents of an abbreviation when no space appears in ink print. A space must be left on either side of an abbreviation in all other situations.

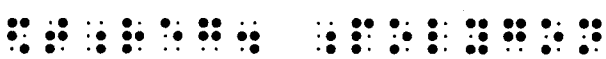
(1) 60 mph = 88 ft/sec

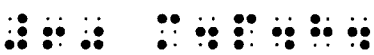


(2) $(3\ \text{yd})^2 = 9\ \text{yd}^2$ 

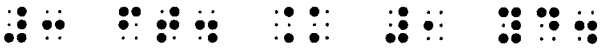
(3) $\sqrt{60\ \text{ft}}$ 

(4) $\frac{1\ \text{hr.}}{60\ \text{min.}}$ 

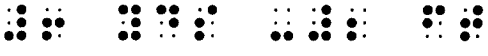
(5) $\triangle_{\text{reg. polygon}}$ 

(6) 60 m.p.h. 

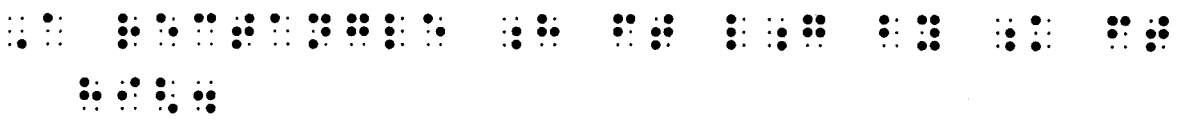
(7) 42 ft. b.m. 

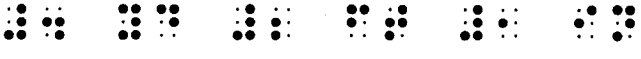
(8) 3 ft. = 1 yd. 

(9) $Q_1 = U_1 - \text{p.e.}$ 

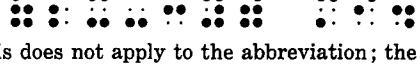
(10) 6 yds — 2 ft 

(11) A rectangle h ft long by k ft high.

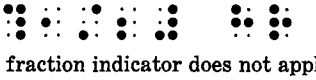


(12) 4 yd 2 ft 1 in 

(13) N 35° W 

(14) (2x — 3y) mi. 

(the closing parenthesis does not apply to the abbreviation; the abbreviation must be preceded by a space)

(15) ½ hr 

(the closing fraction indicator does not apply to the abbreviation; the abbreviation must be preceded by a space)

(16) 25 Sq. Ft. 

RULE IX—CONTRACTIONS AND SHORT-FORM WORDS

§55. Non-Use of Contractions and Short-Form Words:

a. Contractions and short-form words must not be used in a word, part of a word, or abbreviation when it is in direct contact with any item listed below. They must also not be used before or after the space which immediately precedes or follows a sign of comparison. In the case where transition to another braille line has been made, contractions must not be used if they could not have been used without the transition. In the case of an expression containing a hyphen or dash, only that portion between the hyphen or dash and the item with which direct contact is made is subject to this rule.

i. Any braille indicator other than capitalization indicators or the italic sign of English Braille.

(1) 13_{seven}

(2) **4-ampere**

(3) velocity

(4) 

(in ink print, a picture of a church)

(5) (one)

(6) inch-pound^2

(7) $\frac{\text{distance}}{\text{time}} = \text{rate}$

ii. Any numeric symbol written as in the Nemeth Code.

(1) $\cos\left[2 \text{ Arc csc}\left(-\frac{29}{21}\right)\right]$

iii. The general omission symbol.

(1) ten ? four = six

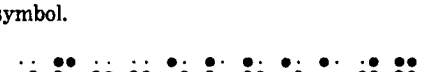
iv. A single letter.

(1) a arc sin x + b arc tan y

v. Any sequence of more than one letter in which each letter has a separate identity.

(1) xy sine z 

vi. Any modifier symbol.

(1) $\xrightarrow{\text{heat}}$ 

vii. The radical symbol.

(1) $\sqrt{\text{four}}$ 

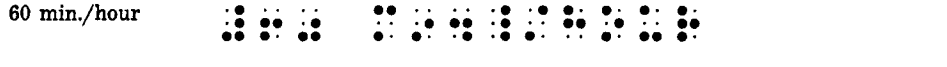
viii. Any operation symbol.

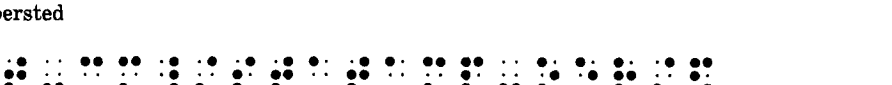
(1) nine — seven = two

(2) ergs/cm^3 

(3) 60 min./hour 

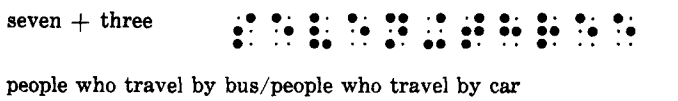
(4) statvolt-cm/statamp-oersted



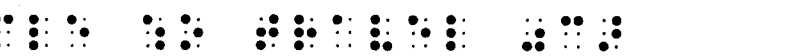
(5) $\frac{\text{distance}}{\text{time}} = \text{rate}$ 

(6) seven + three 

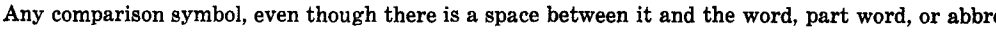
(7) people who travel by bus/people who travel by car



ix. Any comparison symbol, even though there is a space between it and the word, part word, or abbreviation.

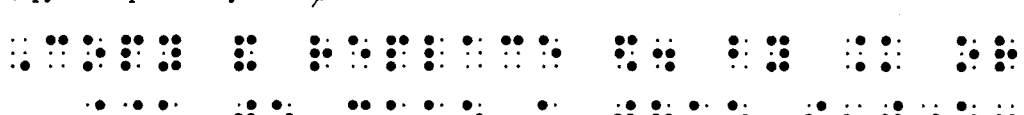
(1) 1 hour = 60 minutes 

(2) Let 3x = the larger number

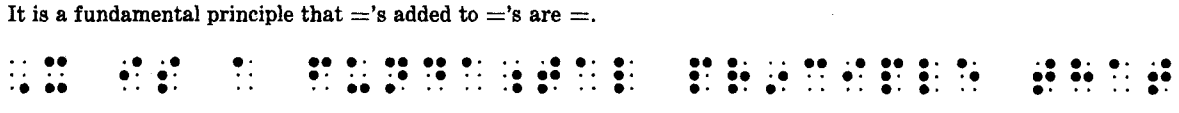


(3) seven + three = ten

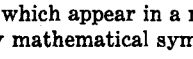
(4) Copy and replace $\square$ by $=$ or $\neq$ to make a true sentence.



(5) It is a fundamental principle that ='s added to ='s are =.



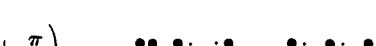
b. Contractions must not be used in abbreviated function names in any context. In addition, contractions must not be used in unabbreviated function names which appear in a mathematical context. In particular, the word "arc" must not be contracted when immediately preceded or followed by mathematical symbols, whether spaced or unspaced.

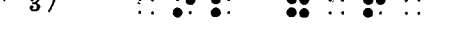
(1) sin x 

(2) cosh x 

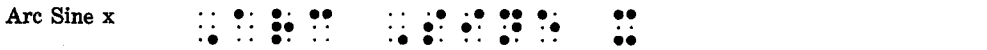
(3) sine x + sine y 

(4) cos $\left(\text{arc Tan x} + \frac{\pi}{3}\right)$ 

(5) 2 arcsin x 

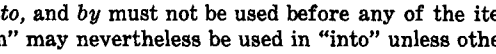
(6) Arc Sine x 

(7) Arc ACB is a major arc.



c. The contractions for *to, into,* and *by* must not be used before any of the items listed below. When the contraction for *into* may not be used, the contraction for "in" may nevertheless be used in "into" unless otherwise prohibited.

i. Before any of the items in a above.

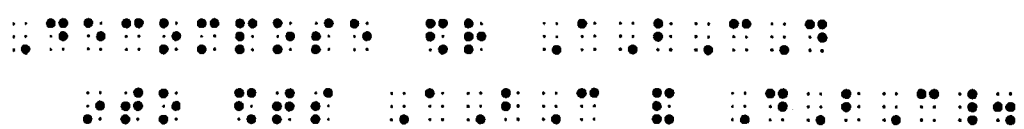
(1) From a to z. 

(contraction not used according to a(i))

(2) From $\frac{1}{4}$ to $\frac{1}{2}$. 

(contraction not used according to a(i))

(3) Decompose ▭ ABCD into ◬ ABC and DBC.



(contraction not used according to a(i))

(4) From α to ω. 

(contraction not used according to a(i))

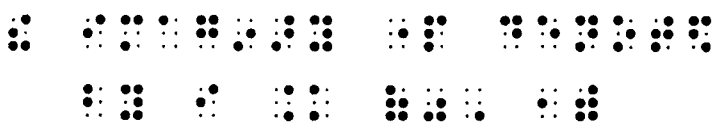
(5) From —10 to 10. 

(contraction not used according to a(ii))

(6) 22×3 is equal to $? \times 11$

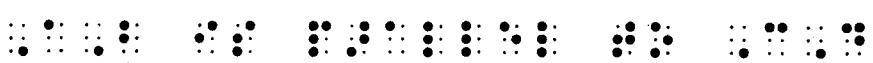
(contraction not used according to a(iii))

(7) the imaginary part denoted by $i = (0, 1)$

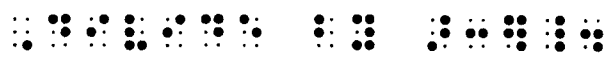


(contraction not used according to a(iv))

(8) AB is parallel to CD



(contraction not used according to a(v))

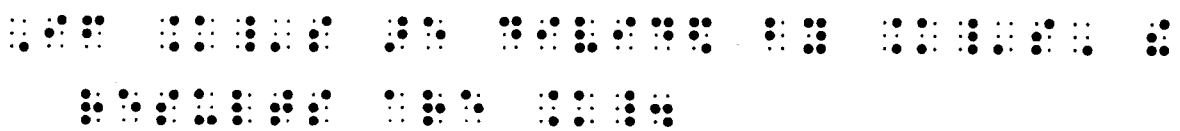
(9) Divide by $\sqrt{3}$. 

(contraction not used according to a(vii))

(10) From — 10 to +10. 

(contraction not used according to a(viii))

(11) If ='s are divided by ='s, the results are =.

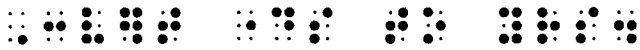


(contraction not used according to a(ix))

ii. Before any abbreviation which consists of one letter or a combination of letters corresponding to a short-form word.

(1) Turn to p. 27. 

(2) Convert mm to m. 

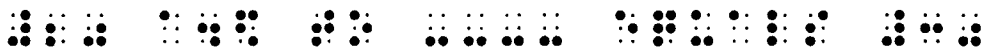
(3) Convert days to yrs. 

iii. Before any Roman numeral.

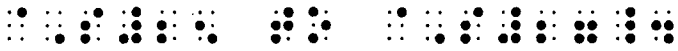
(1) Chapters I to VII. 

iv. Before a dash or ellipsis.

(1) 20 added to ____ equals 30

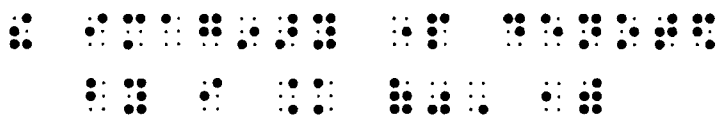


v. Before any reference symbol.

(1) §25 to §27. 

vi. Before a "single letter".

(1) the imaginary part denoted by i = (0, 1)



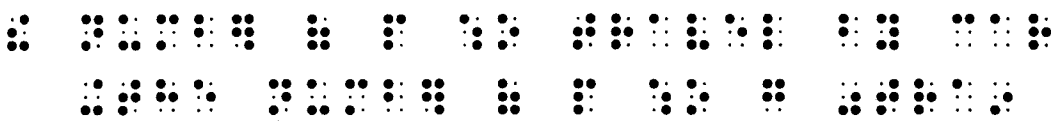
vii. Before a sequence of more than one letter in which each letter has a separate identity.

(1) AB is parallel to CD

viii. Before any word, part word, or abbreviation in situations in which contractions are not permitted according to any of the other rules of this section.

(1) people who go by car + people who go by train

(2) the number of people who travel by car + the number of people who go by train

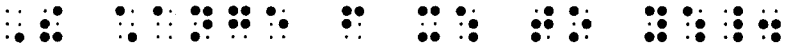


("by" cannot be contracted before "car" because the *ar* contraction in "car" cannot be used)

(3) The area is divided into in^2.

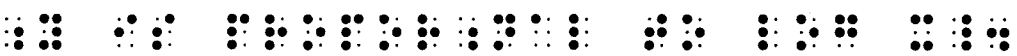
ix. Before any modified expression.

(1) The change from $\overline{x}$ to $\overline{y}$.



x. Before any abbreviated function name or unabbreviated function name, provided that the latter occurs in a mathematical context.

(1) y is proportional to log x.



xi. Before any grouping symbol.

(1) From (1) to (5).

xii. Before any of the miscellaneous symbols of Rule XXII.

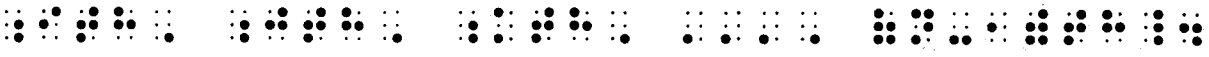
(1) Change to %.

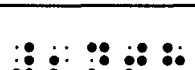
(2) Change the money into $10 bills.

d. The *st* and *th* contractions must not be used for ordinal endings when these are attached to numerals, letters, or other mathematical expressions. If an ordinal ending is composed of only one letter, follow the ink print.

(1) 1st, 2nd, 3rd, 4th.

(2) ith, jth, kth, . . . , (n-1)th.



(3) 2nth 

(4) 1st and 2d. 

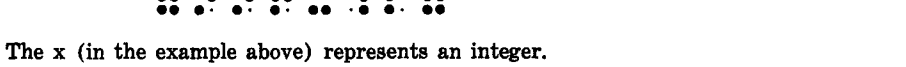
e. The one-cell whole word alphabet contractions for *but, can, . . . , you, as* and the one-cell lower-sign whole-word contractions for *be, enough, were, his, in, was,* whether capitalized, italicized, or neither, must not be used when these words are in direct contact with any grouping symbol. The contractions, whole-word or part-word, for *and, for, of, the, with,* whether capitalized, italicized, or neither, must also not be used when in direct contact with any grouping symbol. If any punctuation intervenes between a grouping symbol and any contraction of the types mentioned above, the rule still applies. When this rule precludes the use of a contraction in one part of a word, no part of the word may be contracted.

(1) (and, in addition) 

(2) (that is) 

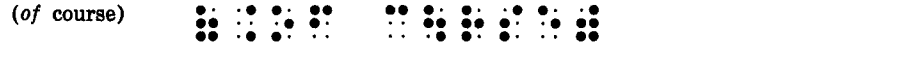
(3) (not-p) 

(4) The x (in the example above) represents an integer.



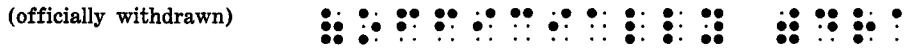
(5) (*of* course) 

(6) (Formally a polynomial)



(7) (officially withdrawn)

(8) ("Can you find the answer?")



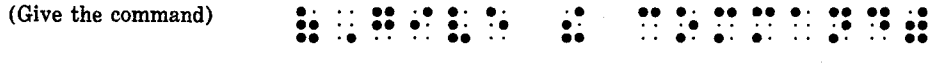
(9) ("in addition, $x \neq 0$")

(10) ("Of course not!") 

(11) (Give the command) 

f. Contractions must not be used when they are likely to be mistaken for mathematical expressions.

(1) Use the $\int$ to find the volume.



(2) Can C = 100?

(3) a = b, but b ≠ c.

(4) We see that c = d.

(5) Let *this* be an angle in standard position.

("this" is spelled out because in immediately surrounding text in ink print the Greek letter "theta" (θ) also appeared)

§56. Use of Contractions and Short-Form Words: Subject to the conditions of §55, the use of contractions and short-form words of English Braille must be used.

(1) 1 light-year

(2) not-p

(3) x-intercept

(4) unary, binary, . . . , m-ary.

(5) ½-off sale

(6) 9-inch

(7) hydrogen-3

(8) 360°-interval

(9) ?-ounce

(the omission symbol replaces a question mark in ink print)

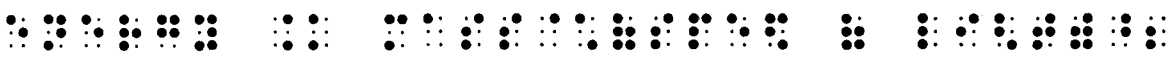
(10) (ft.-pound)

(11) inch-pound^2

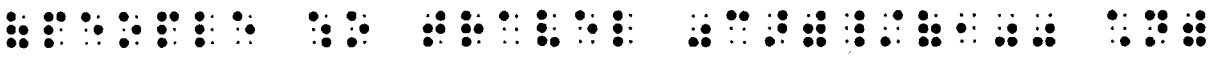
(12) 60 min./hour

(13) (rate) × (time) = (distance)

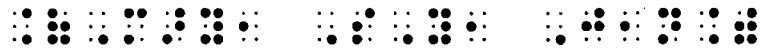
(14) energy = mass × (speed of light)^2



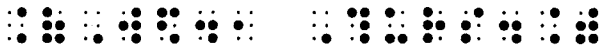
(15) (people who travel by car)/(100 children)



(16) {Mary, Sally, Jean}



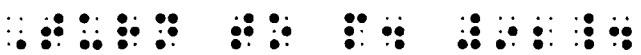
(17) {Wed., Thurs.}



(18) The hypothesis about ab is untenable.

(19) The abbreviation for "sine" is "sin."

(20) Turn to p. 92.

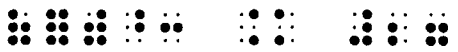


RULE X—OMISSIONS

General Omission Symbol

§57. **Omissions:** A large number of signs are employed in ink print to denote omitted mathematical or literary material. When a question mark, either by itself or in combination with hyphens or dashes, or a blank space is employed in ink print to denote omission, the general omission symbol must be used in the transcription. The number of general omission symbols to be used must be the same as the omission signs in ink print. When a dash is used to denote omission in ink print, the long dash must be used in the transcription. If an omission sign is used in ink print for which this Code provides no representation, this sign may be represented by drawing it in, or the transcriber may devise a braille symbol to represent it. In all other cases, the omission symbol which is used must correspond to the sign which appears in ink print. All of these rules apply unless work is spatially arranged for computation, in which case see §58.

(1) $(?)^8 = 27$

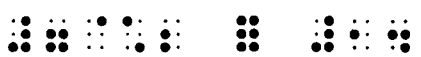


(question mark occurs in ink print)

(2) 92 in. = ? ft. ? in.

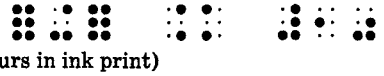
(question mark occurs in ink print)

(3) $7 \times 2 \; ? \; 14$



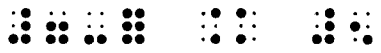
(question mark occurs in ink print)

(4) $? + ? = 10$



(question mark occurs in ink print)

(5) $7 - \underline{?} = 5$



(a dash occurs beneath a question mark in ink print)

(6) 9 — 5 = -?-

(a question mark is preceded and followed by a hyphen in ink print)

(7) (5,) + (, 15) = (7, 13)

(blank spaces occur in ink print)

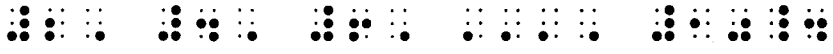
(8) 5 × 25 =

(a blank space occurs in ink print)

(9) five × —— = fifteen

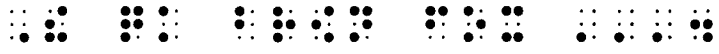
(a dash occurs in ink print)

(10) 2, 4, 6, . . . , 10.



(an ellipsis occurs in ink print)

(11) The quick brown fox . . . .



(an ellipsis occurs in ink print)

(12) □ + ○ = 5

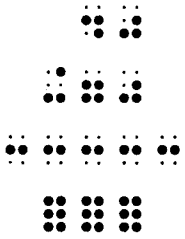
(a square and circle occur in ink print)

(13) 5 × 7 ~ 35

(an extended tilde occurs in ink print)

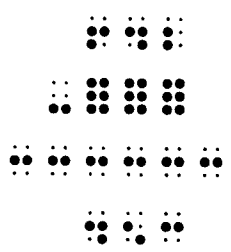
§58. **Omissions in Work Arranged Spatially for Computation:** In work arranged spatially for computation, only the general omission symbol may be used in braille regardless of how the omission is denoted in ink print. In addition, the number of general omission symbols to be used must be the same as the number of omission signs which occur in ink print.

(1)
40
+70
???



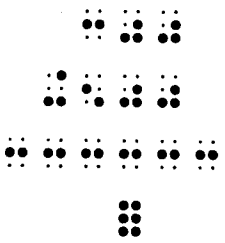
(question marks are shown in ink print)

(2)
642
—???
453



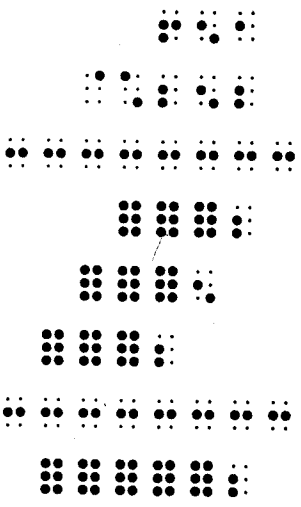
(question marks are shown in ink print)

(3)
300
+500
?



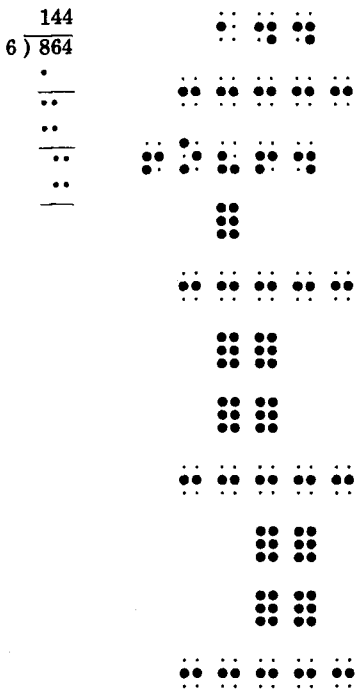
(a question mark is shown in ink print)

(4)
651
×252
•••2
•••5
•••2
•••••2



(dots are shown in ink print)

(5) 144
6) 864



(dots are shown in ink print)

§59. **Spacing with Omissions:** The general omission symbol should be spaced in the same manner as the material which it replaces. Other omission symbols must be spaced in accordance with the rules governing the spacing of those symbols.

RULE XI—CANCELLATION

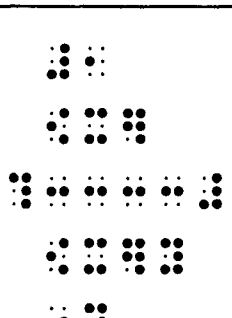
Cancellation Indicators

Opening 

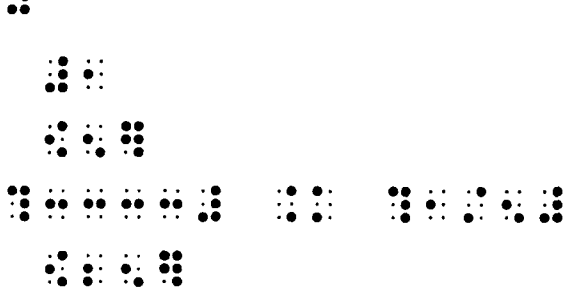
Closing 

§60. **Cancellation Indicators:** The cancellation indicators must be used to show the extent of a mathematical expression which has been canceled in ink print. A spatial arrangement must be used when cancellation is represented in braille. Whenever a fraction or any of its parts is canceled, a spatial arrangement must be used for that fraction. Items which are individually canceled in ink print must be represented as individually canceled in the transcription.

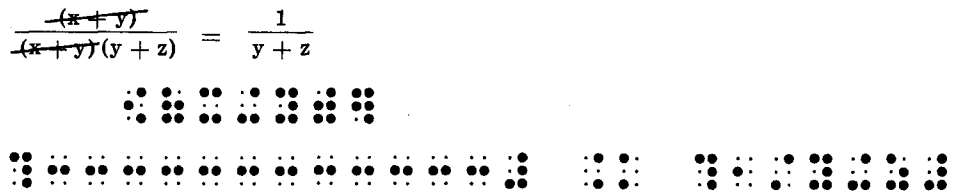
(1) $\frac{1}{\cancel{x}\,\cancel{xy}\,y}$



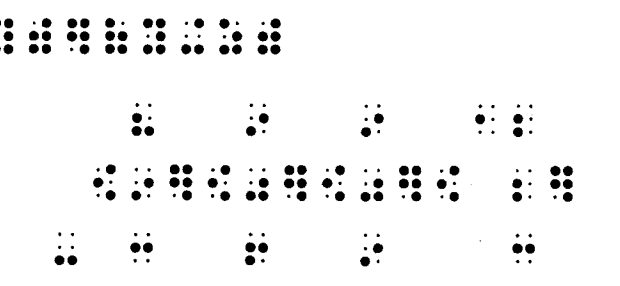
(2) $\frac{1}{\cancel{5}\,\cancel{25}\,5} = \frac{1}{5}$



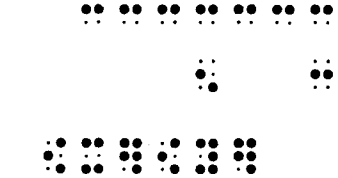
(3) $\frac{\cancel{(x+y)}}{\cancel{(x+y)}(y+z)} = \frac{1}{y+z}$



(4) $\begin{array}{r} \overset{8}{\cancel{8}}\,\overset{9}{\cancel{9}}\,\overset{9}{\cancel{9}}\,\overset{12}{\cancel{2}} \\ -\ 3\ 6\ 9\ 3 \\ \hline 5\ 3\ 0\ 9 \end{array}$



(5) $\frac{\cancel{xy}}{\cancel{xy}z}$



RULE XII—FRACTIONS

Fracton Indicators

Simple

Opening 

Closing 

Complex

Opening 

Closing 

Hypercomplex

Opening 

Closing 

Fractional Part of a Mixed Number

Opening 

Closing 

Fraction Lines

Used with Simple-Fraction Indicators

Diagonal line or slash / 

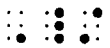
Horizontal — 

Used with the Fractional Part of a Mixed Number

Diagonal line or slash / 

Horizontal — 

Used with Complex-Fraction Indicators

Diagonal line or slash / 

Horizontal — 

Used with Hypercomplex-Fraction Indicators

Horizontal —— 

Used with a Spatial Arrangement

Horizontal (varying in length) —— 

§61. Simple Fractions: For the purposes of this Code, a *simple fraction* is one whose numerator and denominator contain no fractions except possibly at the superscript or subscript level.

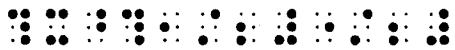
§62. Use of Simple-Fraction Indicators:

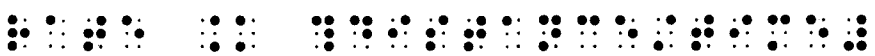
a. Simple-fraction indicators must be used, except in the case of mixed numbers, to enclose a simple fraction whose numerator and denominator are separated by a horizontal fraction line in ink print.

(1) $\frac{1}{3}$ 

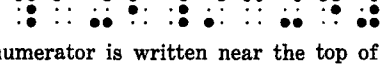
(2) $x^{\frac{1}{2}}$ 

(3) $\frac{a+b}{c}$ 

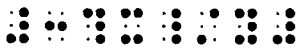
(4) $\frac{x^{\frac{1}{2}}}{2}$ 

(5) $\text{rate} = \frac{\text{distance}}{\text{time}}$ 

b. Simple-fraction indicators must be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in ink print, when the expressions on either side of the diagonal line appear at different levels relative to it, or in different type size than is normal for the purpose for which these expressions are used.

(1) $a + b/c + d$ 

(in ink print, the numerator is written near the top of the diagonal line and the denominator is written near the bottom)

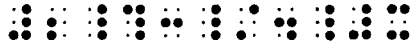
(2) $3^{x/y}$ 

(in ink print the 3, x, and y are at the same level, but the x and y are in smaller type than the 3)

§63. Non-Use of Simple-Fraction Indicators:

a. Simple-fraction indicators must not be used to enclose the fractional part of a mixed number.

(1) $4\frac{3}{8}$ 

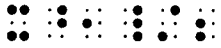
(2) 2 3/4 x 

b. Simple-fraction indicators must not be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in ink print when the expressions on either side of the diagonal line appear at the same level relative to it, or are of the

same type size as the surrounding mathematical text. Sometimes the expressions on either side of the diagonal line are not the terms of a fraction at all. Even when they are, the transcriber cannot always be certain of where the fraction begins or ends. Accordingly, it is better to avoid the use of indicators altogether in these cases and permit the braille reader to make a judgment based on the same information that is available to the sighted reader.

(1) 1/3 

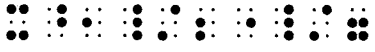
(in ink print, 1 and 3 are at the same level)

(2) $x^{1/2}$ 

(in ink print, 1 and 2 are at the same level; although the 1 and 2 are in smaller type, they are of normal size for printing superscripts)

(3) $x_{1/2}$ 

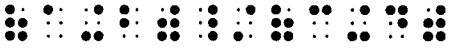
(in ink print, the x and 2 are at the same level and are of normal size for printing base-line signs)

(4) $x^{1/2/7}$ 

(in ink print, 1 and 2 are at the same level and x and 7 are at the same level; each pair of signs is of normal size for printing at its respective level)

(5) a + b/c + d 

(in ink print, all letters are of normal size and at the same level on either side of a diagonal line)

(6) (a + b)/(c + d) 

(in ink print, all letters are of normal size and at the same level on either side of a diagonal line)

(7) I/(n) 

(the expressions on either side of the diagonal line are not the terms of a fraction)

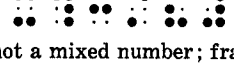
(8) 1/31/70 

(the expression represents a date)

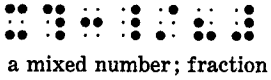
§64. **Mixed Numbers:** For the purposes of this Code, a mixed number is an expression which begins with a numeral and is followed, usually in smaller type, by a simple fraction whose numerator and denominator are both numerals. The fraction line of this simple fraction may be either horizontal or diagonal in ink print. The mixed-number indicators must be used to enclose the fractional part of a mixed number. An expression is not a mixed number if it contains any letter, even though such an expression is of the same form as a mixed number in every other respect.

(1) $4\frac{3}{8}$ 

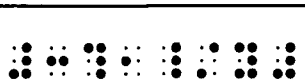
(2) 4 3/8 

(3) $x\frac{3}{8}$ 

(this is not a mixed number; fraction in smaller type than the x)

(4) $x_{3/8}$ 

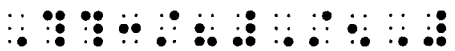
(this is not a mixed number; fraction in smaller type than the x)

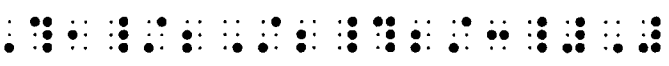
(5) 3 1/y 

(this is not a mixed number; fraction in smaller type than the 3)

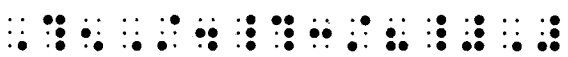
§65. Complex Fractions: For the purposes of this Code, a complex fraction is one whose numerator, denominator, or both, contains at least one simple fraction. A fraction is not a complex fraction if the only simple fractions it contains are at the superscript or subscript level.

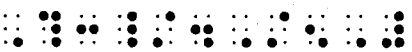
§66. Use of Complex-Fraction Indicators: Complex-fraction indicators must be used to enclose a complex fraction.

(1) $\frac{\frac{3}{8}}{5}$ 

(2) $\frac{1/2}{2\frac{2}{3}}$ 

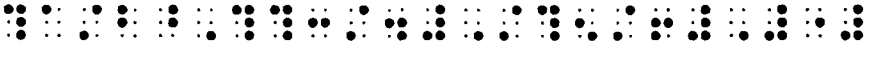
(3) $\frac{2/3}{3/2}$ 

(4) $\frac{5}{4\frac{3}{8}}$ 

(5) $\frac{3/4}{5}$ 

(6) $\frac{1}{2}/\frac{3}{4}$ 

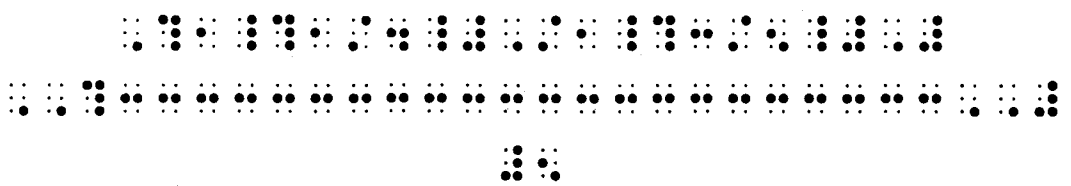
§67. Hypercomplex Fractions: For the purposes of this Code, a hypercomplex fraction is one whose numerator, denominator, or both, contain at least one complex fraction. A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level.

(1) $\frac{a}{b^{\frac{\frac{3}{4}}{\frac{5}{6}}}}$ 

(this is not a hypercomplex fraction)

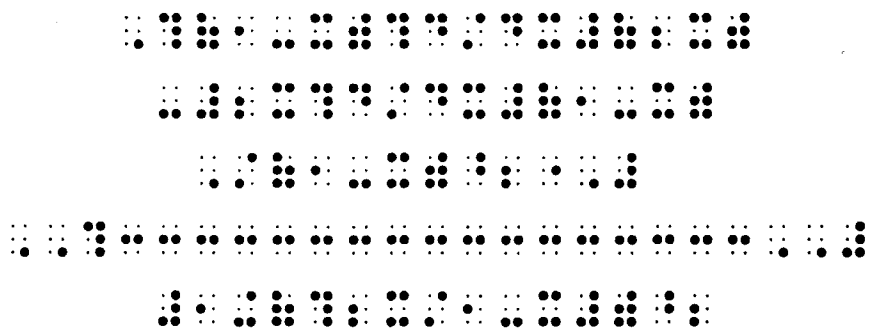
§68. Use of Hypercomplex-Fraction Indicators:

a. Hypercomplex-fraction indicators must be used to enclose a hypercomplex fraction. The use of a linear arrangement within a spatial arrangement is preferable to an arrangement which is entirely linear or entirely spatial.

(1) $\frac{1\frac{1}{4}}{\frac{1\frac{3}{5}}{5}}$ 

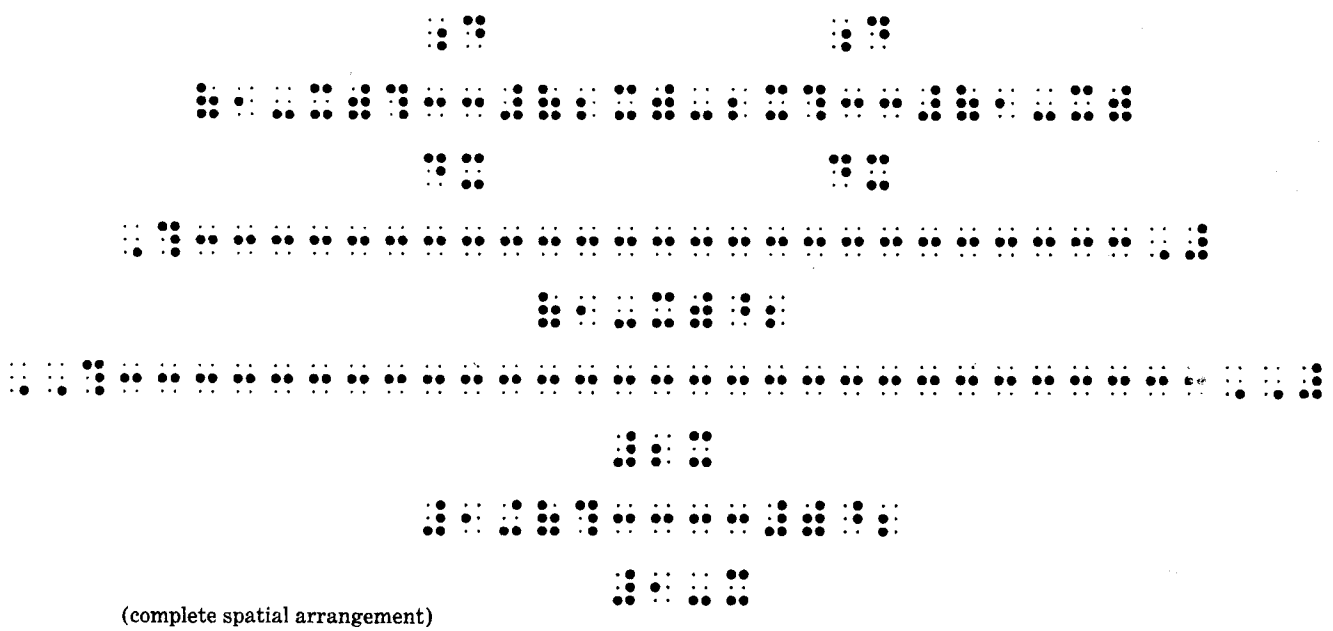
(preferred method of transcribing a hypercomplex fraction)

(2) $$\frac{\dfrac{(1-x)\,\frac{d}{dx}\,(2x) - 2x\,\frac{d}{dx}\,(1-x)}{(1-x)^2}}{1 + \left(\dfrac{2x}{1-x}\right)^2}$$



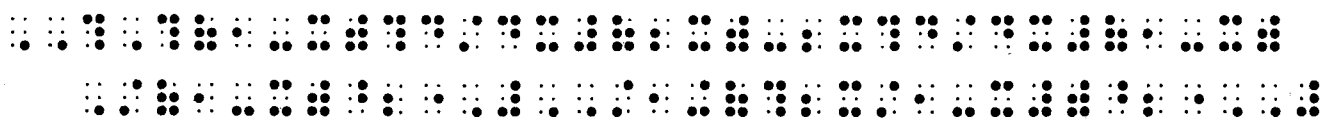
(preferred method of transcribing a hypercomplex fraction)

(3) $$\frac{\dfrac{(1-x)\,\frac{d}{dx}\,(2x) - 2x\,\frac{d}{dx}\,(1-x)}{(1-x)^2}}{1 + \left(\dfrac{2x}{1-x}\right)^2}$$



(complete spatial arrangement)

(4) $$\frac{(1-x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(1-x)}{\frac{(1-x)^2}{1+\left(\frac{2x}{1-x}\right)^2}}$$

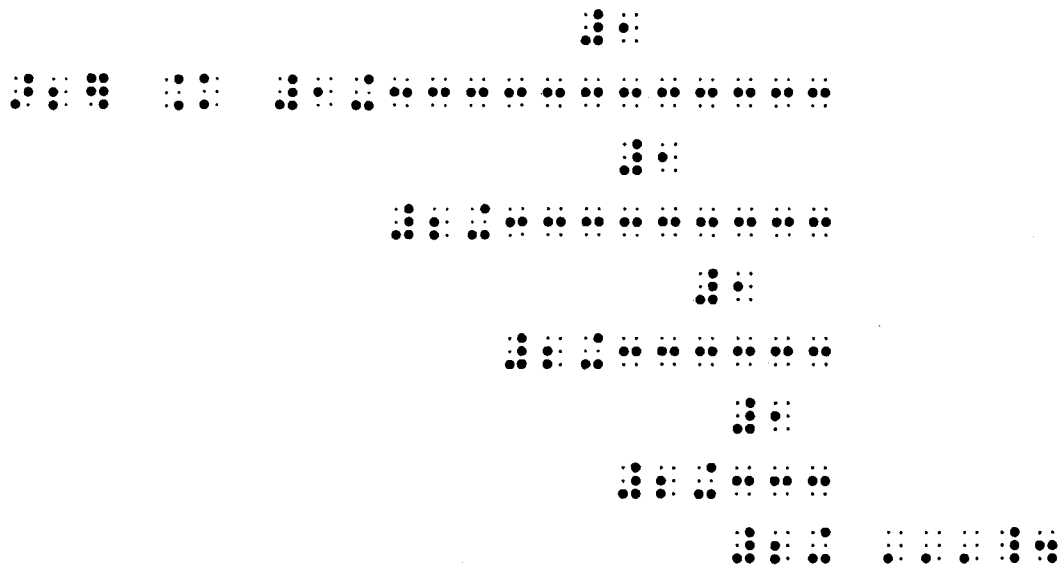


(complete linear arrangement)

b. Hypercomplex fractions of higher order may be transcribed in the manner suggested by a above. It is only necessary to use dot 6 the proper number of times before the fraction indicators and their matching fraction line.

§69. Continued Fractions: A continued fraction is one in which each denominator, except possibly the last one, is the sum of a whole number and a fraction. A spatial arrangement must be used for each fraction. In this case, each fraction line must have proportionately the length shown in print, and fraction indicators of any kind must not be used with a continued fraction.

(1) $$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots}}}}$$



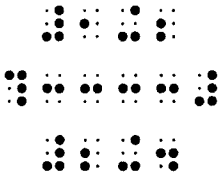
§70. Spatial Arrangement for Fractions:

a. Whenever a fraction is transcribed spatially, all fraction indicators, except for continued fractions, must be shown, and each fraction line must have precisely the length necessary to cover the longest expression to which it applies, and the terms of the fraction

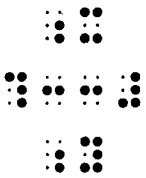
must be centered on their fraction lines. The runover of an expression which is too long to be centered on the fraction line which applies to it may be effected at suitable places in accordance with the rules for runovers, but each portion of the divided expression must be centered on the fraction line to which the expression, as a whole, applies.

b. In general, the linear arrangement for fractions must be used when not expressly forbidden in the case of continued fractions. However, when fractional notation is first presented to the reader, as in the lower grades, or when there is any other special need, any fraction may be represented spatially.

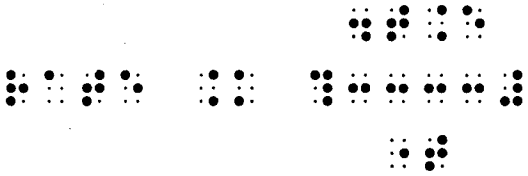
(1) $\frac{1+2}{2+4}$



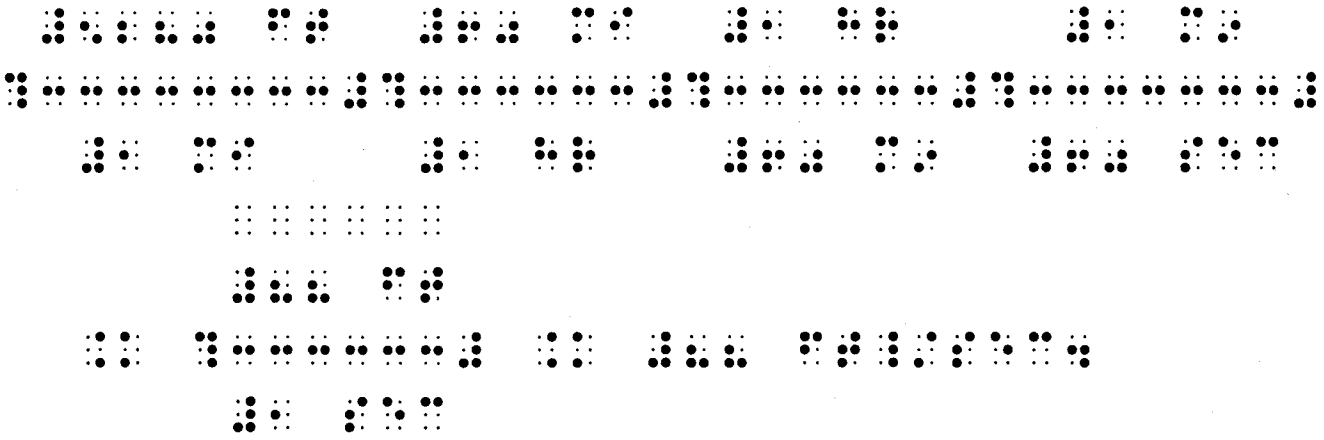
(2) $\frac{x}{y}$



(3) $\text{rate} = \frac{\text{distance}}{\text{time}}$



(4) $\frac{5280\text{ ft}}{1\text{ mi}}\ \frac{60\text{ mi}}{1\text{ hr}}\ \frac{1\text{ hr}}{60\text{ min}}\ \frac{1\text{ min}}{60\text{ sec}} = \frac{88\text{ ft}}{1\text{ sec}} = 88\text{ ft/sec.}$



c. For spatial arrangement of fractions in connection with cancellation see §60.

d. For spatial arrangement of hypercomplex fractions see §68.

e. For spatial arrangement of continued fractions see §69.

RULE XIII—SUPERSCRIPTS AND SUBSCRIPTS

Level Indicators

Base Line

Superscript

Superscript with Superscript

Superscript with Subscript

Superscript with Superscript with Superscript

Superscript with Superscript with Subscript

Superscript with Subscript with Superscript

Superscript with Subscript with Subscript

Subscript

Subscript with Superscript

Subscript with Subscript

Subscript with Superscript with Superscript

Subscript with Superscript with Subscript

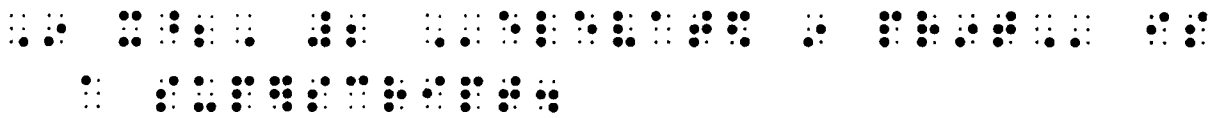
Subscript with Subscript with Superscript

Subscript with Subscript with Subscript

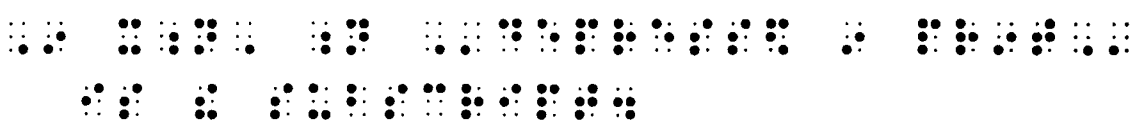
Contraction for Comma and Optional Space at Superscript or Subscript Level

§71. Nature of Superscripts and Subscripts: It is characteristic of mathematical expressions to employ signs, usually in smaller type, which are elevated or depressed relative to the *base line*. A sign which is elevated relative to the base line is called a *superscript;* one which is depressed relative to the base line is called a *subscript*. When an entire expression is at the superscript or subscript level, it should be written without an indicator in braille, but its position must be explained to the reader by a transcriber's note.

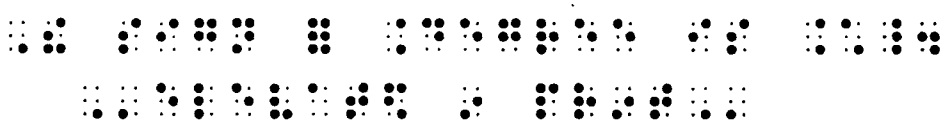
(1) In x^2, ^{2} is a superscript.



(2) In x_n, $_n$ is the subscript.



(3) The sign for *degree* is °.



§72. Hierarchy of Superscripts and Subscripts: Superscripts or subscripts may carry superscripts or subscripts of their own; the latter are then referred to as superscripts or subscripts of *second order*, and are thus distinguished from the former, which are called superscripts or subscripts of *first order*. Second order superscripts or subscripts may, in turn, carry superscripts or subscripts of their own, which are then called superscripts or subscripts of *third order*. While it is theoretically possible for a superscript or subscript to be of order higher than the third, this situation rarely arises in practice.

§73. Level Indicators: A level indicator other than the base-line indicator identifies the symbols which follow it as representing a superscript or subscript. The *base-line* indicator identifies the symbols which follow it as representing signs on the base line. The degree of elevation or depression specified by a level indicator is *always with respect to the base line;* the symbol which precedes the indicator, if it represents a sign at some other level, plays no role in this regard.

§74. Orientation by Level Indicator:

a. The effect of a level indicator with one component is to direct the reader's attention upward or downward from the base line according as that component is the superscript or the subscript indicator.

(1) x^2 

(x squared)

(2) y^3 

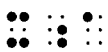
(y cubed)

(3) x^* 

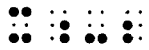
(x carries an asterisk as a superscript)

(4) x^{-2} 

(x carries minus 2 as a superscript)

(5) x_a 

(x sub a)

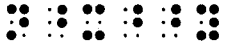
(6) x_{-2} 

(x carries minus 2 as a subscript)

b. The effect of a level indicator with two components may be analyzed as follows:

i. The first component directs the reader's attention upward or downward from the base line according as that component is, in itself, the superscript or subscript indicator.

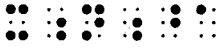
ii. The second component then directs the reader's attention upward or downward from this new position according as the second component is, in itself, the superscript or subscript indicator.

(1) n^{x^y} 

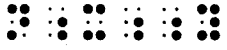
(n carries a superscript x which carries a superscript y)

(2) x^{n_a} 

(x carries a superscript n which carries a subscript a)

(3) x_{n^a} 

(x carries a subscript n which carries a superscript a)

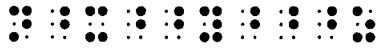
(4) n_{x_y} 

(n carries a subscript x which carries a subscript y)

c. The effect of a level indicator with three components may be analyzed as follows:

i. The first two components direct the reader's attention from the base line to the position described in **b** above.

ii. The third component then directs the reader's attention upward or downward from this new position, according as that component is, in itself, the superscript or the subscript indicator.

(1) $n^{x^{y^z}}$ 

(n carries a superscript x which carries a superscript y which carries a superscript z)

(2) $n^{x^{y^{z\cdots}}}$ 

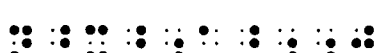
(the ellipsis indicates the presence of superscripts of increasingly higher order; the dots are printed obliquely)

(3) $x^{y^{z_a}}$ 

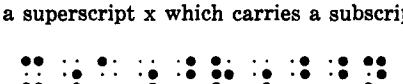
(x carries a superscript y which carries a superscript z which carries a subscript a)

(4) $x^{y_{a^n}}$ 

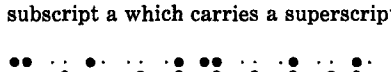
(x carries a superscript y which carries a subscript a which carries a superscript n)

(5) $n^{x_{aj}}$ 

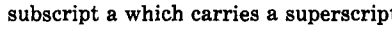
(n carries a superscript x which carries a subscript a which carries a subscript j)

(6) $x_{a^{rn}}$ 

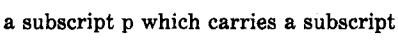
(x carries a subscript a which carries a superscript r which carries a superscript n)

(7) $x_{a^{n_b}}$ 

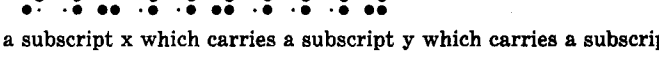
(x carries a subscript a which carries a superscript n which carries a subscript b)

(8) $x_{p_{am}}$ 

(x carries a subscript p which carries a subscript a which carries a superscript m)

(9) $n_{x_{y_z}}$ 

(n carries a subscript x which carries a subscript y which carries a subscript z)

(10) $n_{x_{y_z}}$ 

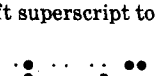
(the ellipsis indicates the presence of subscripts of increasingly higher order; the dots are printed obliquely)

d. The effect of a level indicator with more than three components may be analyzed in the same manner suggested for level indicators with two or three components.

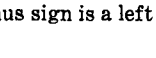
§75. Left Superscripts and Subscripts: A superscript or subscript may occupy a position to the left, as well as to the right, of the sign to which it applies. The words *left* or *right* are then used with the words *superscript* or *subscript* to make the distinction in position.

A right or left superscript or subscript is represented as such merely by preserving the relative horizontal positions of the superscript or subscript symbol and the symbol to which it applies. Each must be preceded by its appropriate level indicator.

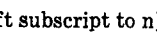
Left superscripts or subscripts of the third or higher order, although rare, are treated in the manner suggested by the examples below.

(1) ^{x}n 

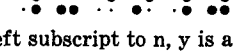
(x is a left superscript to n)

(2) ^{-}x 

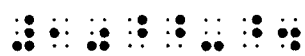
(the minus sign is a left superscript to x)

(3) $_{x}n$ 

(x is a left subscript to n)

(4) $_{x}n_{y}$ 

(x is a left subscript to n, y is a right subscript to n)

(5) 10^{-4} 

(10 to the minus 4 power; the minus is superscript to the 4)

(6) ${}^{n_a}x$ 

(n sub a is a left superscript to x)

(7) ${}^{an}x$ 

(a is a left subscript to n, the combination is a left superscript to x)

(8) ${}^{n^a}x$ 

(n to the a power is a left subscript to x)

(9) ${}^{a}_{n}x$ 

(a is a left superscript to n, the combination is a left subscript to x)

(10) ${}^{x}_{y}n$ 

(x sub y is a left subscript to n)

(11) ${}_{y}{}^{x}n$ 

(y is a left subscript to x, the combination is a left subscript to n)

(12) ${}^{b\ c}p\,x$ 

(p carries a right superscript b and c is a left superscript to x)

§76. **Direct Superscripts and Subscripts:** A superscript or subscript which occupies, respectively, a position directly over or under the sign to which it applies is called a *modifier* (see **Rule XIV**).

§77. **Numeric Subscripts:** The subscript indicator must not be used to indicate a numeric subscript provided that all of the following conditions hold:

i. The corresponding numeric sign must be a *right*, and not a left, subscript.

ii. The corresponding numeric sign must be a subscript of *first order*, and not of higher order.

iii. The sign with which the numeric subscript is associated must be an abbreviated function name or a letter which has a separate identity. In the latter case, this letter must not be any letter which represents a numeral in a non-decimal base. Otherwise, the letter may be from any alphabet and in any type form, and may be modified by one or more primes, or a superscript. In the case of a two-letter abbreviation for a chemical compound, the abbreviation must be treated as if it were a letter.

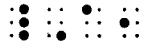
iv. The subscript must consist of numeric symbols only, and must carry no superscripts or subscripts of its own.

(1) x_1 

(x sub 1; subscript indicator not required because all conditions i-iv hold)

(2) x_{11} 

(x sub 1 1; subscript indicator not required because all conditions i-iv hold)

(3) $\mathfrak{A}_1$ 

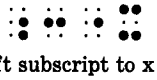
(German capitalized ah sub 1; subscript indicator not required because all conditions i-iv hold)

(4) x'_1 

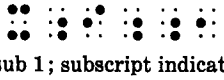
(x prime sub 1; subscript indicator not required because all conditions i-iv hold)

(5) x''_2 

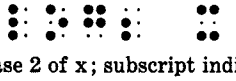
(x double prime sub 2; subscript indicator not required because all conditions i-iv hold)

(6) ${}_3x$ 

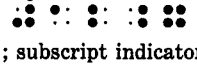
(3 is a left subscript to x; subscript indicator is required because condition i does not hold)

(7) x_{i_1} 

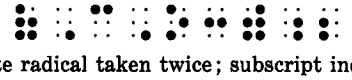
(x sub i sub 1; subscript indicator is required because condition ii does not hold)

(8) $\log_2 x$ 

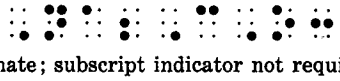
(log to the base 2 of x; subscript indicator not required because all conditions i-iv hold)

(9) 12_7 

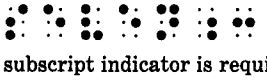
(12 sub 7; subscript indicator is required because condition iii does not hold)

(10) $(CO_3)_2$ 

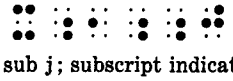
(the carbonate radical taken twice; subscript indicator is required before the 2 because condition iii does not hold)

(11) Na_2CO_3 

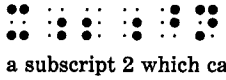
(sodium carbonate; subscript indicator not required because all conditions i-iv hold)

(12) $seven_3$ 

(seven sub 3; subscript indicator is required because condition iii does not hold)

(13) x_{1_j} 

(x sub 1 sub j; subscript indicator is required because condition iv does not hold)

(14) x_{2^n} 

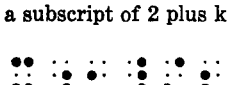
(x carries a subscript 2 which carries a superscript n; subscript indicator is required because condition iv does not hold)

(15) $x_{2'}$ 

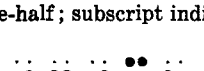
(x sub 2 prime; subscript indicator is required because condition iv does not hold)

(16) x_{2+k} 

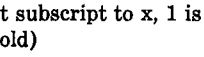
(x carries a subscript of 2 plus k; subscript indicator is required because condition iv does not hold)

(17) $x_{1/2}$ 

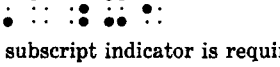
(x sub one-half; subscript indicator is required because condition iv does not hold)

(18) ${}_3x_1$ 

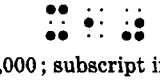
(3 is a left subscript to x, 1 is a right subscript to x; subscript indicator is required before the 3 because condition i does not hold)

(19) A_{x1} 

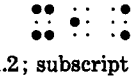
(A sub x 1; subscript indicator is required because condition iv does not hold)

(20) $x_{10,000}$ 

(x sub 10,000; subscript indicator not required because all conditions i-iv hold)

(21) $x_{1.2}$ 

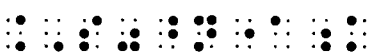
(x sub 1.2; subscript indicator not required because all conditions i-iv hold)

(22) $x_{.6}$ 

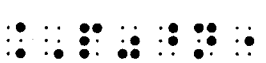
(x sub .6; subscript indicator not required because all conditions i-iv hold)

(23) $\sum_0^n a_k$ 

(the summation from zero to n of a sub k; subscript indicator is not required because all conditions i-iv hold)

(24) $\Pi_0^n a_k$ 

(the product from zero to n of a sub k; subscript indicator is not required because all conditions i-iv hold)

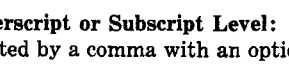
(25) $3AF_{16}$ 

(A and F represent a numeral in base 16; subscript indicator is required because condition iii does not hold)

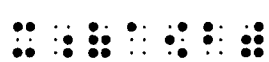
(26) $\int_0^{\sqrt{1-x^2}} f(x)\ dx$ 

(the integral from 0 to the square root of 1 — x^2 of f of x dx; subscript indicator is required because condition iii does not hold)

§78. Comma at Superscript or Subscript Level: A commonly occurring superscript or subscript notation is the one in which two consecutive items are separated by a comma with an optional space following the comma. In this configuration, the symbol (dots 2-4-6) must be used to replace the comma and the optional space used in this way. This contracted form must not be used to replace a comma and the optional space which follows it in a configuration which is on the base line.

(1) $x_{i,\,j,\,k}$ 

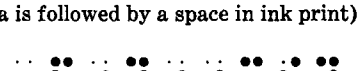
(each comma is followed by a space in ink print)

(2) $x_{(a,b)}$ 

(the comma is not followed by a space in ink print)

(3) $x_{1,2}$

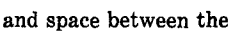
(the comma is followed by a space in ink print)

(4) $P_{n_{x,y}}$ 

(the comma is followed by a space in ink print)

(5) $x_{n-1,\,n-1}$, $x_{n-1,\,n}$, $x_{n,\,n-1}$

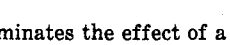
(the comma and space between the items on the base line cannot be contracted)

(6) (x, y) 

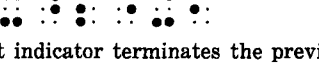
(the comma and space between the items on the base line cannot be contracted)

§79. Circumstances Determining Changes of Level: The symbols and situations listed below have the following effect in determining changes of level.

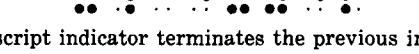
a. A level indicator terminates the effect of a previous level indicator and initiates the level implied by the new indicator. In the case of the base-line level, the previous base-line indicator may only be implied.

(1) $x^2 + 1$ 

(the superscript indicator terminates the previous implied base-line level and initiates the superscript level, the base-line indicator terminates the previous superscript level and initiates the base-line level)

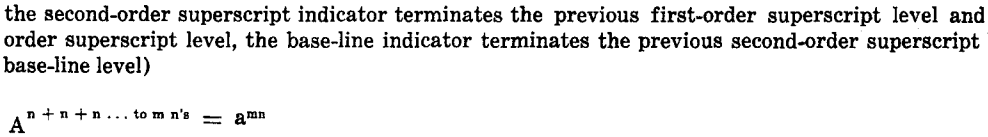
(2) $x_a + y^2$ 

(the subscript indicator terminates the previous implied base-line level and initiates the subscript level, the base-line indicator terminates the previous subscript level and initiates the base-line level, the superscript indicator terminates the previous base-line level and initiates the superscript level)

(3) $\frac{e^{x^2}}{2}$ 

(the superscript indicator terminates the previous implied base-line level and initiates the first-order superscript level, the second-order superscript indicator terminates the previous first-order superscript level and initiates the second-order superscript level, the base-line indicator terminates the previous second-order superscript level and initiates the base-line level)

(4) $A^{n+n+n\ldots\text{to m n's}} = a^{mn}$



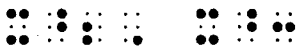
(the superscript indicator which follows the n preserves the effect of the preceding superscript indicator; otherwise, the punctuation indicator would terminate the effect of the previous level indicator and initiate the base-line level)

b. The punctuation indicator terminates the effect of any previous level indicator and initiates the base-line level. In addition, the comma, provided it is not a numeric symbol, terminates the effect of any previous indicator and initiates the base-line level. However, the comma, when it is a numeric symbol and the contracted form ⠠ (dots 2-4-6), preserves the level that is already in effect.

(1) x^2.

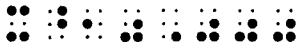
(the period is at the base-line level)

(2) x^2, x^3



(the comma is at the base-line level)

(3) $x^{10,000}$



(the comma is a numeric symbol and preserves the superscript level)

(4) $x_{i,\,j}$

(the contracted form for a comma and optional space preserves the subscript level that is already in effect)

(5) $P_{n_1,\,n_2,\,\ldots}$

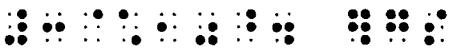
(the contracted form for a comma and optional space preserves the indicated subscript level)

c. A space or the transition to a new braille line which is followed by literary text or unrelated mathematical text terminates the effect of any previous level indicator and initiates the base-line level. However, if a space occurs between the parts of an abbreviation or phrase, the appropriate level indicator must be restated before each part.

(1) $2p^2$ is always even.

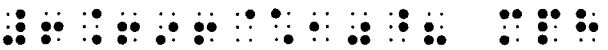
(the space before the literary text terminates the previous superscript level and initiates the base-line level)

(2) 3×10^4 ergs



(the space before the literary text terminates the previous superscript level and initiates the base-line level)

(3) 6.696×10^8 mph

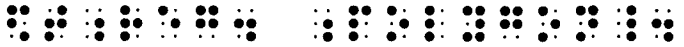


(the space before the literary text terminates the previous superscript level and initiates the base-line level)

(4) $(x^2\ y^2)$

(these items are entries in a matrix and hence unrelated; the space terminates the previous superscript level and initiates the base-line level)

(5) $\triangle_{\text{reg. polygon}}$



(level indicators are required between each part of this abbreviation to show that they are both at the subscript level)

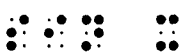
(6) $\triangle_{\text{regular polygon}}$

(level indicators are required between each part of this phrase to show that they are both at the subscript level)

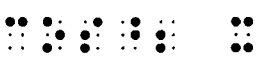
d. The space which immediately follows a symbol of shape, an abbreviated function name, or an unabbreviated function name, provided the latter is in a mathematical context, preserves the level that is already in effect. If these items carry a superscript or subscript, the space which follows such a superscript or subscript reinstates the level that was in effect before.

(1) $b_{\triangle ABC}$ 

(the space preserves the subscript level at which the triangle appears)

(2) sin x 

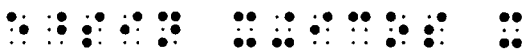
(the space preserves the base-line level of sin)

(3) $\cos^2 x$ 

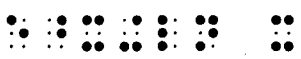
(the space reinstates the base-line level of cos)

(4) $e^{\sin x}$ 

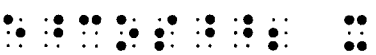
(the space preserves the superscript level at which sin appears)

(5) $e^{\sin x + i \cos x}$ 

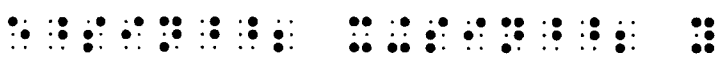
(each space preserves the superscript level at which the abbreviated function names appear)

(6) $e^{x + \ln x}$ 

(the space preserves the superscript level at which ln appears)

(7) $e^{\cos^2 x}$ 

(the space reinstates the superscript level at which cos appears)

(8) $e^{\sin^2 x + \sin^2 y}$ 

(each space reinstates the superscript level at which the abbreviated function names appear)

(9) $q^{\log_q a}$ 

(the space reinstates the superscript level at which log appears)

(10) $V_{max\ (m,\ n)}$ 

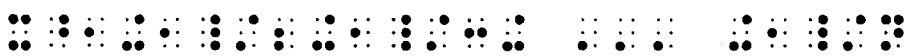
(the space preserves the subscript level at which max appears)

e. The space which occurs in a numeral for the purpose of dividing it into short regular segments preserves the level already in effect.

(1) $e^{3.14159\ 26535}$ 

f. The space which precedes an ellipsis or long dash preserves the effect of any previous level indicator. The space which follows the ellipsis or long dash preserves the level that is already in effect. However, if such a space is followed by literary text, unrelated mathematical text, or a sign of comparison, this space initiates the base-line level.

(1) $x^{1} + 1/2 + 1/3 + \ldots + 1/n$

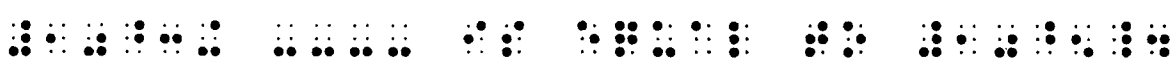


(both spaces preserve the superscript level)

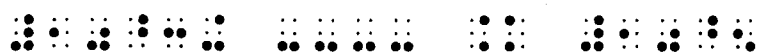
(2) $s_1 \ldots s_n$ 

(both spaces preserve the implied base-line level)

(3) $10^{3} + \text{—}$ is equal to 10^{5}.



(the space before the dash preserves the previous superscript level, and the space following the dash terminates the effect of the previous superscript level and initiates the base-line level)

(4) $10^{3} + \text{—} = 10^{5}$ 

(the space before the dash preserves the previous superscript level, and the space following the dash terminates the effect of the previous superscript level and initiates the base-line level)

g. The space or transition to a new braille line which is followed by a comparison symbol terminates the effect of a level indicator already in effect and initiates the base-line level. The space after a comparison symbol preserves the level that is already in effect.

(1) $x^2 + y^2 + z^2 = r^2$ 

(the space which is followed by the equals symbol terminates the effect of the preceding superscript level and initiates the base-line level, the space after the equals symbol preserves the base-line level)

(2) $2^x < 3^x$ 

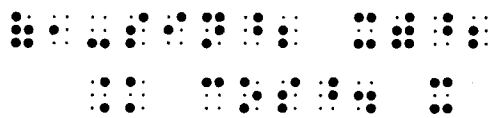
(the space which is followed by the less than symbol terminates the effect of the preceding superscript level and initiates the base-line level, the space after the less than symbol preserves the base-line level)

(3) $q^{\log_q a} = a$ 

(the space which is followed by the equals symbol terminates the effect of the preceding superscript level and initiates the base-line level, the space after the equals symbol preserves the base-line level)

(4) $\int_{u=a}$ 

(the subscript indicator before the equals symbol keeps this symbol at the subscript level; the space after the equals symbol preserves the level that is already in effect)

(5) $(1 - \sin^2 x)^2 = \cos^4 x$ 

(the transition to a new braille line before the equals symbol terminates the previous superscript level and initiates the base-line level)

h. Any symbol or situation other than those in a to g preserves the level that is already in effect.

§80. Use of Level Indicators:

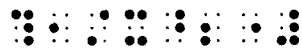
a. A level indicator must be used before any braille indicator or grouping symbol whenever this braille indicator or grouping symbol applies to a level other than the one currently in effect.

(1) $\sqrt{x^2 + y^2}$ 

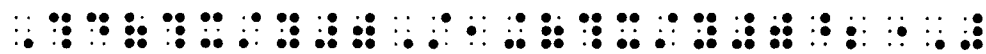
(the termination indicator applies to the base line, therefore the base-line indicator is required)

(2) $e^{\sqrt{x^2 + y^2}}$ 

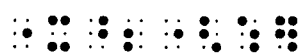
(the termination indicator applies to the first-order superscript level, therefore the superscript indicator is required)

(3) $\frac{1}{x^2}$ 

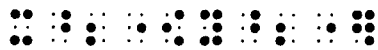
(the closing simple-fraction indicator applies to the base line, therefore the base-line indicator is required)

(4) $\dfrac{d\left(\frac{x}{y}\right)}{1 + \left(\frac{x}{y}\right)^2}$ 

(the closing complex-fraction indicator applies to the base line, therefore the base-line indicator is required)

(5) $\overline{x^2}$ 

(the directly-over indicator applies to the base line, therefore the base-line indicator is required)

(6) $x^2y^{\not 2}$ 

(the opening and closing cancellation indicators apply to the base line, therefore the base-line indicator is required)

(7) $(x^2 + y^2)$ 

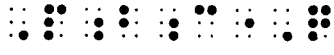
(the closing parenthesis applies to the base line, therefore the base-line indicator is required)

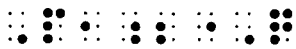
(8) $x^{(m^n)}$ 

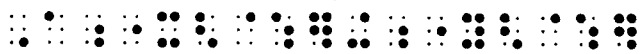
(the closing parenthesis applies to the first-order superscript level, therefore the superscript indicator is required)

b. The superscript indicator must be used to restate the superscript level when two superscripts are consecutive but one applies to the expression which precedes it and the other applies to the expression which follows it. Similarly, the subscript indicator must be restated when two subscripts are consecutive and one applies to the expression preceding it and the other applies to the expression following it. A superscript or subscript indicator must be restated before a modified expression which is interior to the superscript or subscript expression, provided that the multipurpose indicator is also used.

(1) $p^b\ ^cq$ 

(2) $P_b\ _cQ$ 

(3) $P_1\ _2Q$ 

(4) $A_{\widetilde{x}+\widetilde{y}}$ 

(the subscript indicator after the plus sign must be restated before the multipurpose indicator)

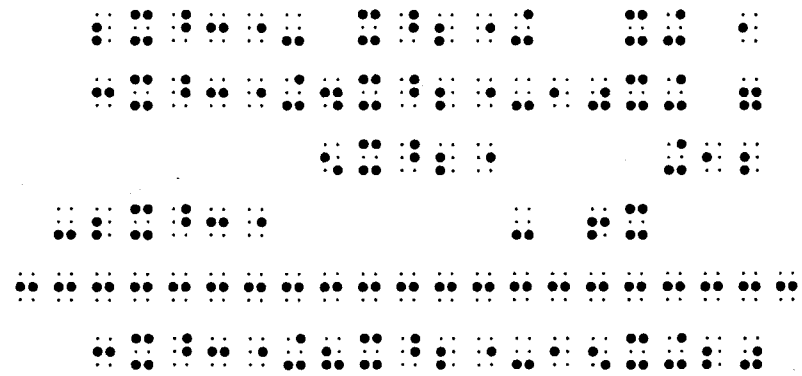
c. The appropriate level indicator must be used before each part of an abbreviation or phrase which is at a level other than the base line.

(1) $\triangle_{\text{regular polygon}}$

(2) $a^{n+n+n \ldots \text{to m n's}}$

d. Whenever spaces are left for the purpose of achieving alignment, level indicators must be used as though such spaces were not present.

(1)
$$\begin{array}{rrrrrr} 2x^3 & - & x^2 & + & x & + & 1 \\ 3x^3 & + & 4x^2 & - & 10x & + & 7 \\ & & 5x^2 & & & + & 12 \\ - & 2x^3 & & - & 6x & & \\ \hline 3x^3 & + & 8x^2 & - & 15x & + & 20 \end{array}$$



e. The appropriate level indicator must be used before any symbol or situation in which a change of level is required but the change is not effected by any of the conditions of §79.

(1) 360°-interval

(the base-line indicator places the hyphen at the base-line level)

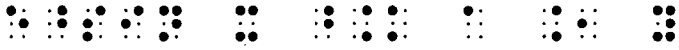
(2) $\int_{u=a}$

(the subscript indicator before the equals symbol keeps this symbol at the subscript level; otherwise it would be at the base-line level)

(3) $t\Big]_{t=a}^{t=b} = b - a$

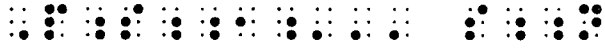
(the subscript and superscript indicators before the first two equals symbols keep these at the subscript and superscript levels respectively, while the space before the last equals symbol places it at the base-line level)

(4) $e^{\sin x = a} > y$



(the superscript indicator before the equals symbol keeps this symbol at the superscript level)

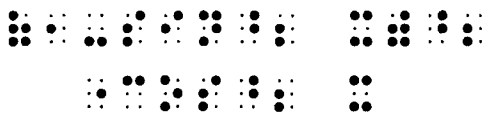
(5) $P_{s_1 \ldots s_n}$



(the subscript indicator before the ellipsis places the ellipsis at the first-order subscript level)

(6) $P_1^{\alpha_1} \ \ldots \ P_r^{\alpha_r}$ 

(the base-line indicator places the ellipsis at the base-line level)

(7) $(1 - \sin^2 x)^2 \cos^2 x$ 

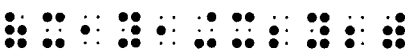
(it is assumed that this expression has had to be run over to another braille line at the place indicated; the base-line indicator at the beginning of the braille line places cos at the base-line level; otherwise it would have remained at the superscript level initiated on the previous braille line)

§81. Non-Use of Level Indicators:

a. The base-line indicator must not be used to return to the base line from a numeric subscript if the subscript indicator has not been used before the numeric subscript.

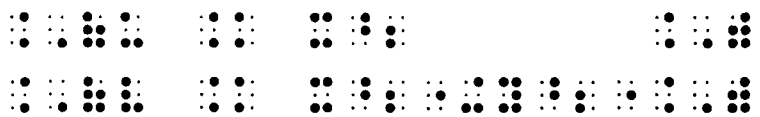
(1) $(x_1 + 1)$ 

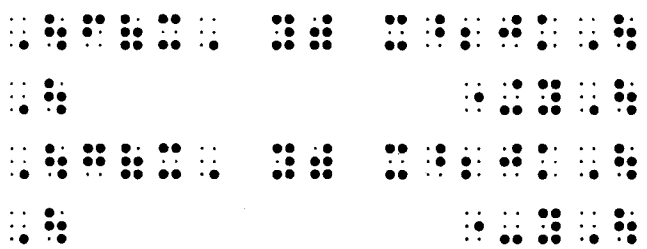
(base-line indicator not required before the plus symbol)

(2) $(x_1y_1 + x_2y_2)$ 

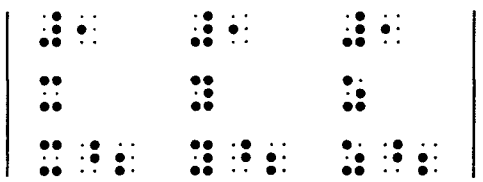
(base-line indicator not required after any of the numeric subscripts)

b. The base-line indicator must not be used before a right enlarged grouping symbol if this symbol either is separated from its preceding material by one or more spaces, or if the material which precedes the right grouping symbol is not the end of an expression.

(1) $\left\{ \begin{matrix} u = x^2 \\ v = x^2 + y^2 \end{matrix} \right\}$ 

(2) $\left| \begin{matrix} f(x, y) & x^{2jk} + y \\ g(x, y) & x^{2jk} - y \end{matrix} \right|$ 

c. A level indicator must not be used before any closing grouping symbol which is drawn in.

(1) $\left| \begin{matrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{matrix} \right|$ 

d. A level indicator must not be used to change the level if any symbol or situation specified in §79 has already effected the change to the desired level.

§82. Simultaneous and Non-Simultaneous Superscripts and Subscripts:

a. When an expression simultaneously carries a superscript and subscript, the subscript must be indicated first, even if the subscript is numeric and does not require the subscript indicator. However, if this sign carries one or more primes in addition, see §83.

(1) x_a^n 

(x carries simultaneously a subscript of a and a superscript of n)

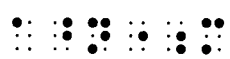
(2) ${}_a^n x$ 

(x carries simultaneously a left subscript of a and a left superscript of n)

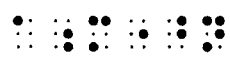
(3) x_1^2 

(x carries simultaneously a subscript of 1 and a superscript of 2)

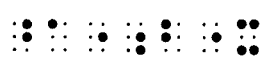
b. When the same expression carries a superscript and a subscript which are not simultaneous, the relative horizontal positions of the signs must be retained in the transcription, but the base-line indicator must be inserted before making the transition to the other level.

(1) a^n_m 

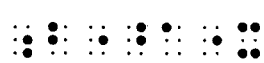
(the superscript is closer to the a than the subscript)

(2) $a_m{}^n$ 

(the subscript is closer to the a than the superscript)

(3) ${}^a_b x$ 

(the left subscript is closer to the x than the left superscript)

(4) ${}_b{}^a x$ 

(the left superscript is closer to the x than the left subscript)

(5) $x_1{}^2$ 

(the subscript is closer to the x than the superscript)

(6) $x_a'^b$ 

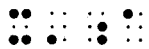
(the subscript is closer to x prime than the superscript)

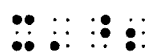
§83. Primes in Addition to Superscripts or Subscripts:

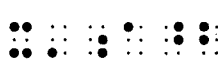
a. The prime symbol must never be preceded by the superscript indicator.

(1) x' 

b. When an expression carries one or more primes in addition to superscripts or subscripts, the prime symbol or symbols must be indicated first unless such symbols do not occur at the beginning of the superscript or subscript, in which case they must retain the same position as in ink print.

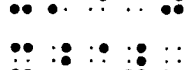
(1) x'_a 

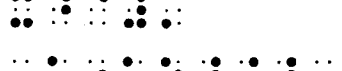
(2) x'^2 

(3) x'^b_a 

(4) x''^3_1 

(5) x'^* 

(6) $x^{*\prime}$ 

(7) $A^{*\prime}_{ue}$ 

(8) A'^*_{ue} 

c. For primes in other roles see §172.

§84. **Plurals and Possessives:** For plurals or possessives of mathematical expressions which end with a superscript or subscript see §39.

RULE XIV—MODIFIERS

Modification Indicators

Directly Over

First order 

Second order 

Directly Under

First order 

Second order 

Multipurpose 

Superposition 

Termination 

Modifiers

Arc

Concave upward

Concave downward

Arrow

Barbed at both ends

Barbed at left

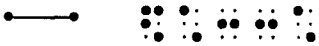
Barbed at left and dotted at right

Barbed at right

Contracted form

Uncontracted form

Dotted at both ends

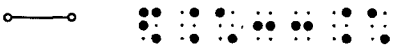


Dotted at left (no barb)

Dotted at left and barbed at right

Dotted at right (no barb)

Hollow dot at both ends



Hollow dot at left (no barb)

Hollow dot at left and barbed at right

Hollow dot at right and barbed at left

Hollow dot at right (no barb)

Bar

Horizontal (macron)

Vertical

Caret (circumflex) ^

Inverted v

Left-pointing <

Right-pointing >

Dot •

Hollow Dot °

Question Mark ?

Tilde

Extended ≈

Simple ~

Triangle (equilateral) △

§85. Modifiers: A modifier is a superscript or subscript which occupies, respectively, a position directly over or directly under the sign to which it applies. The modifiers in the list at the beginning of this rule are those most commonly used, but other modifiers must be treated in the same manner.

§86. Modified Expressions:

a. The Five-Step Rule for Transcribing Modified Expressions: The components of a modified expression must appear in the following order:

i. Multipurpose indicator

ii. Expression being modified.

iii. Directly-over indicator or directly-under indicator

iv. Modifier.

v. Termination indicator

These five components may never be separated from each other by transition to another braille line. The termination indicator terminates only the modified expression; it does not affect the level at which the modified expression occurs.

(1) $\underline{x}$

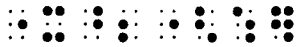
(x with subscribed bar)

(2) $\overline{x+y}$ 

(x plus y, with superscribed bar)

(3) $\lim_{x \to 0} f(x)$ 

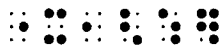
(the limit, as x approaches 0, of f of x)

(4) $\overline{x^2}$ 

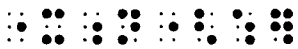
(x squared, with superscribed bar)

(5) $\overline{x'}$ 

(x primed, with superscribed bar)

(6) $\overline{x_1}$ 

(x sub 1, with superscribed bar)

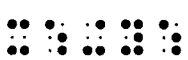
(7) $\overline{x_n}$ 

(x sub n, with superscribed bar)

b. When the expression being modified is a single digit or a letter, lower-case or capitalized, from any alphabet, and in any type form, and when the modifier is the horizontal bar directly above such a single digit or letter, the digit or letter, followed by the bar, serves to express the modification. This construction should be regarded as a contracted form of expression and must be used whenever applicable. If the modification includes a superscript, subscript, or prime, the five-step rule of a above must be followed. The five-step rule may be used in conjunction with the contracted form without fear of confusion.

(1) $\bar{x}$ 

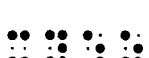
(x with superscribed bar)

(2) $\bar{x} + \bar{y}$ 

(x with superscribed bar plus y with superscribed bar)

(3) $\bar{x}y$ 

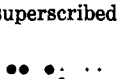
(x with superscribed bar, times y)

(4) $x\bar{y}z$ 

(x, times y with superscribed bar, times z)

(5) $\bar{x}^2$

(x with superscribed bar squared)

(6) $\bar{x}'$ 

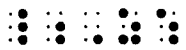
(x with superscribed bar primed)

(7) $\overline{x}_1$ 

(x with superscribed bar, sub 1)

(8) $\overline{x}_n$ 

(x with superscribed bar, sub n)

(9) $\overline{\mathbf{Z}}$ 

(boldface capital Z with superscribed bar)

(10) $3.5\overline{4}$ 

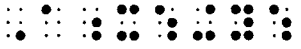
(3.54, bar superscribed to the 4)

(11) $\overline{(\bar{a}A + \bar{b}B)}$ 

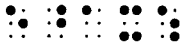
(a with superscribed bar times boldface capitalized A plus b with superscribed bar times boldface capitalized B, the whole expression with superscribed bar)

(12) $A_{\bar{x}}$ 

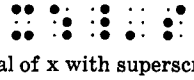
(A with a right subscript of x with superscribed bar)

(13) $A_{\bar{x}+\bar{y}}$ 

(A with a right subscript of x with superscribed bar plus y with superscribed bar)

(14) $e^{a\bar{x}}$ 

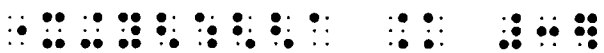
(e with a right superscript of a times x with superscribed bar)

(15) $\overline{x}$'s 

(the plural of x with superscribed bar)

§87. Modifiers of Higher Order:

a. A modifier of the second order must be preceded by the second-order directly-over or directly-under indicator, and a modifier of the third order must be preceded by the third-order directly-over or directly-under indicator. The termination indicator, however, must be used only once, after the last modifier symbol.

(1) $\dfrac{\overline{a=3}}{x+y}$ 

(x plus y superscribed by a bar, which in turn is superscribed by a equals 3)

(2) $\dfrac{x+y}{\underline{a=3}\ \underline{b=2}}$ 

(x plus y subscribed by a bar, which in turn is subscribed by a equals 3, which in turn is subscribed by b equals 2)

b. A modifier of order higher than the third must be treated in the manner suggested in a above.

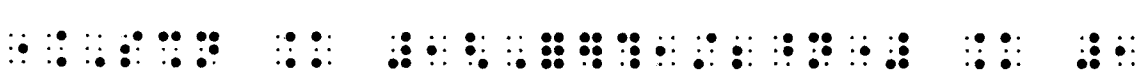
c. A modifier, to be of order higher than the first, must be associated with the same expression as a modifier of lower order. In §86b(11) above, the long bar is not a modifier of second order because no modifier of first order is associated with the same expression as the long bar.

§88. **Simultaneous Modifiers:** When a mathematical expression is simultaneously modified above and below, the modifier below must be indicated first. The termination indicator, however, must be used only once, after the last modifier symbol. If the modifiers involved are of order higher than the first, they are treated as described in §87.

(1) $\overline{\underline{x + y}}$

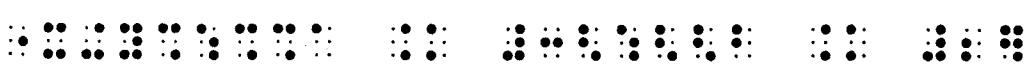
(x plus y, with subscribed and superscribed bars)

(2) $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$



(the Greek capitalized sigma with subscribed n equals 1 and superscribed infinity sign)

(3) $\overline{\underset{a=3}{\underline{x + y}}}^{b=2}$



(x plus y subscribed by a bar which is in turn subscribed by a equals 3; superscribed by a bar which is in turn superscribed by b equals 2)

§89. **Parallel Horizontal Bars:** Parallel horizontal bars must not be regarded as the equals sign or the identity sign when they occur above or below a mathematical expression other than a comparison sign. Furthermore, the bar which is more remote from the mathematical expression being modified must not be regarded as a modifier of second or third order; the double or triple bar must be regarded as a single modifier.

(1) $\overline{\overline{x}}$

(x superscribed by double horizontal bars)

(2) $\overline{\underline{\underline{x}}}$

(x subscribed and superscribed by double horizontal bars)

(3) $\overline{\underline{\underline{x}}}$

(x subscribed by double horizontal bars and superscribed by single horizontal bar)

(4) $\underline{\underline{\underline{x}}}$

(x subscribed by triple horizontal bars)

§90. **Binomial Coefficient:** The two expressions which constitute a binomial coefficient must be separated by the directly-under indicator. The expression which follows the opening parenthesis and precedes the directly-under indicator corresponds to the upper sign

in the binomial coefficient; the expression which follows the directly-under indicator and precedes the closing parenthesis corresponds to the lower sign of the binomial coefficient.

(1) $\binom{n}{k}$ 

(the binomial coefficient with n as the upper sign and k as the lower sign)

(2) $\binom{g_j}{a_j}$ 

(the binomial coefficient with g sub j as the upper sign and a sub j as the lower sign)

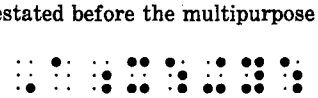
§91. Modified Expressions in Superscripts and Subscripts: If a modified expression is part or all of a right superscript or subscript, the multipurpose indicator must be preceded by the appropriate level indicator. This will automatically be the case if the modified expression occurs at the beginning of the superscript or subscript; but the appropriate level indicator must be restated if the modified expression occurs at an interior position of the superscript or subscript. If the contracted form for a modified expression is used so that the multipurpose indicator does not appear, the appropriate level indicator must not be restated.

(1) $A_{\tilde{x}}$ 

(A carries a subscript of x with superscribed tilde)

(2) $A_{\tilde{x}+\tilde{y}}$ 

(A carries a subscript of x with superscribed tilde plus y with superscribed tilde; the subscript level after the plus sign must be restated before the multipurpose indicator)

(3) $A_{\bar{x}+\bar{y}}$ 

(A carries a subscript of x with superscribed bar plus y with superscribed bar)

§92. Plural Modified Expressions: (See §39).

§93. Modification by Superposition: When one sign modifies another by superposition, in deciding which is the basic sign and which is the superposed sign, the following hierarchy, in descending order, should be used as a guide:

i. Integral sign

ii. Operation signs

iii. Bars — horizontal and vertical

iv. Shape signs

v. Comparison signs

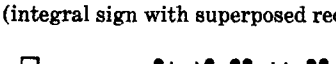
vi. Signs not covered above

A sign belonging to a category lower on the list must be regarded as superposed on a sign higher on the list, and the superposition transcribed accordingly. If two signs belong to the same category, it is permissible to represent the superposition in either

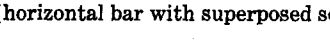
order, provided that the same order is used consistently throughout the entire transcription. The components of a sign compounded by superposition must be joined by the superposition indicator and transcribed unspaced, and without transition to another braille line. The termination indicator must follow the second component. (For other examples, see "Comparison Signs Compounded by Superposition" pages 140-141, and 143.)

(1) 

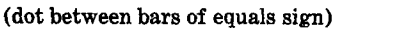
(integral sign with superposed rectangle)

(2) 

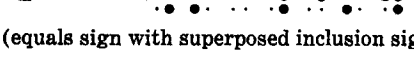
(horizontal bar with superposed square)

(3) 

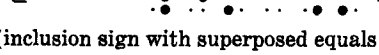
(dot between bars of equals sign)

(4) 

(equals sign with superposed inclusion sign)

(5) 

(inclusion sign with superposed equals sign)

(6) 

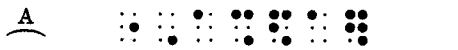
(angle with superposed arc)

§94. Interior Modifiers with Signs of Shape: See §111.

§95. Arc:

(1)

(A with subscribed arc concave upward)

(2) 

(AB with superscribed arc concave upward)

(3) 

(A with subscribed arc concave downward)

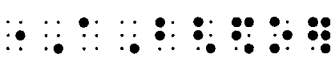
(4) 

(AB with subscribed arc concave downward)

§96. Arrows: Arrows must not be regarded as modifiers when they occur directly over or directly under a comparison sign. In that event, they become a component of a sign of comparison compounded vertically.

When a right-pointing arrow with a single shaft of ordinary length is in regular type, has a full barb, and is not part of a more complex construction or compound modifier, it must be transcribed in its contracted form. If such an arrow is in non-regular type, does not have a full barb or shaft of ordinary length, is part of a compound modifier, or is itself modified, it must be represented in its uncontracted form.

(1) $\overrightarrow{AB}$

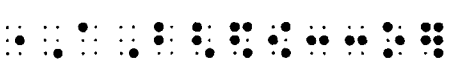


(AB with superscribed arrow barbed at right)

(2) $\overleftarrow{AB}$

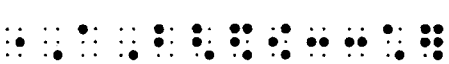
(AB with superscribed arrow barbed at left)

(3) $\overleftrightarrow{AB}$



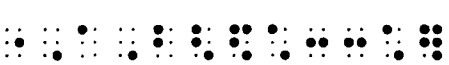
(AB with superscribed arrow barbed at both ends)

(4) $\overleftarrow{AB}$



(AB with superscribed arrow barbed at the left and dotted at the right)

(5) $\overleftarrow{AB}$

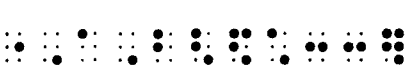


(AB with superscribed arrow dotted at both ends)

(6) $\overleftarrow{AB}$

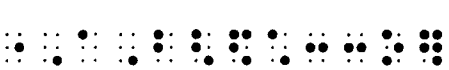
(AB with superscribed arrow with hollow dots at both ends)

(7) $\overleftarrow{AB}$



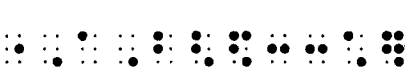
(AB with superscribed arrow dotted at left)

(8) $\overrightarrow{AB}$



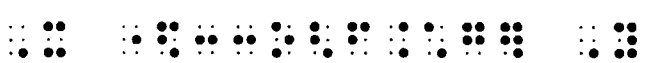
(AB with superscribed arrow dotted at left and barbed at right)

(9) $\overrightarrow{AB}$



(AB with superscribed arrow dotted at right)

(10) $X \xrightarrow{f \circ g} Y$

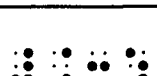


(arrow with superscribed f hollow dot g between X and Y)

§97. Horizontal Bar:

a. The horizontal bar must not be regarded as a modifier when it occurs directly over or directly under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see §147). When the horizontal bar is itself modified by a dot under it or a caret directly over or under it, the combination is a modified sign of comparison (see §146). When the horizontal bar is itself modified by a dot over it, the combination is a sign of operation.

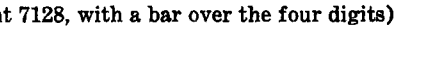
b. The horizontal bar is often used to indicate the recurrence of one or more digits in a decimal numeral by placing it over the digits which recur.

(1) $.\overline{3}$ 

(decimal point 3, with a bar over the 3)

(2) $.\overline{7128}$

(decimal point 7128, with a bar over the four digits)

(3) $3.57\overline{29}$ 

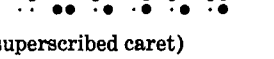
(3.5729, with a bar over the 29)

c. When the horizontal bar occurs over or under the integral sign, or over or under the abbreviated or unabbreviated function name for *limit*, the bar must not be treated as a modifier (see §171 and §118, respectively).

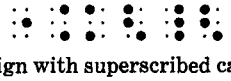
§98. Caret:

(1) $\hat{x}$

(x with superscribed caret)

(2) $\overset{\wedge}{=}$ 

(equals sign with superscribed caret)

(3) $\underset{\vee}{x}$ 

(x with subscribed inverted caret)

(4) $\overset{<}{=}$ 

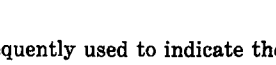
(equals sign with superscribed left-pointing caret)

(5) $\overset{>}{=}$ 

(equals sign with superscribed right-pointing caret)

§99. Dot:

a. The dot is frequently used to indicate the recurrence of one or more digits in a decimal numeral. When used for this purpose, a dot is usually placed in print over each digit of the recurring sequence. In braille, however, only a single dot must be used as a modifier.

(1) $.\dot{3}$ 

(decimal point 3, with a dot over the 3)

(2) .135 

(decimal point 135, with a dot over each of the three digits)

(3) .135 

(decimal point 135, with a dot over the 5)

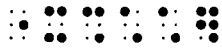
b. Although there is theoretically no limit to the number of dots which may be placed over or under a single mathematical expression, in practice the number rarely exceeds three dots. However, as many dots must be used in the transcription as are present in the printed text, except in the case of recurring decimals as in **a.**

(1) x 

(x with two dots over it)

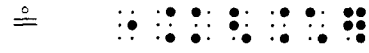
(2) x 

(x with three dots over it)

(3) x 

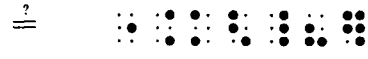
(x with two dots under it)

§100. Hollow Dot:

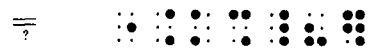
(1) 

(equals sign with superscribed hollow dot)

§101. Question Mark:

(1) 

(equals sign with superscribed question mark)

(2) 

(equals sign with subscribed question mark)

§102. Tilde: The tilde, simple or extended, must not be regarded as a modifier when it occurs directly over or under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see §147). When the tilde, simple or extended, is itself modified by a dot or a caret directly over or under it, the combination is a modified sign of comparison (see §146).

RULE XV—RADICALS

Radical (square root) √ 

Radical Indicators

Index-of-Radical 

Order-of-Radical

First inner radical 

Second inner radical 

Third inner radical 

Termination 

§103. Simple Radicals: The most commonly occurring radical is the square root.

a. When the square root sign has a vinculum (horizontal bar) which specifies the extent to which the radical sign is effective, the transcription of such a radical is accomplished by the following three steps:

i. The radical symbol 

ii. The expression to which it applies (radicand).

iii. The termination indicator 

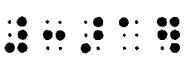
(1) $\sqrt{2}$ 

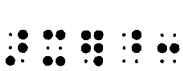
(2) $\sqrt{x+y}$ 

(3) $\sqrt{x^2+1}$ 

(4) $\sqrt{x^2+y^2}$ 

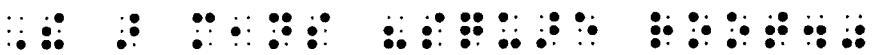
(5) $\sqrt{\frac{x}{y}}$ 

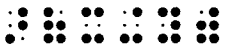
(6) $3\sqrt{a}$ 

(7) $\sqrt{x}^3$ 

b. When the square root sign occurs without a radicand, as when attention is being called to a sign in ink print, or when the extent to which the radical is effective is not indicated in ink print by the vinculum, the termination indicator must be omitted.

(1) The $\surd$ means "square root."



(2) $\surd(x + y)$ 

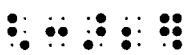
(no vinculum in ink print)

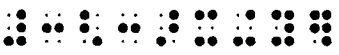
§104. Index of Radical: Radicals of index other than 2 require a specific index. The transcription of such a radical is accomplished by the following three steps:

i. The index-of-radical indicator 

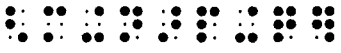
ii. The index of the radical.

iii. Then proceed according to the three steps in §103a.

(1) $\sqrt[3]{2}$ 

(2) $3\sqrt[3]{x + y}$ 

(3) $\sqrt[n]{a}$ 

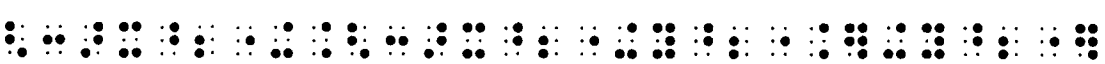
(4) $\sqrt[m+n]{p + q}$ 

§105. Nested Radicals: Occasionally, radicals are nested one within the other. The first inner radical is then regarded as having a depth of order 1, the second inner radical as having a depth of order 2, and so on. In such cases, the order-of-radical indicator (dots 4-6) must be repeated before both the radical symbol and its associated termination indicator as many times as it is necessary to indicate the depth of that radical. If one of the inner radicals is associated with an index, the proper number of order-of-radical indicators must be placed before the index-of-radical indicator rather than before the radical symbol itself. The order-of-radical indicators are provided for the purpose of enabling the reader to keep track of the depth of the radical to which it applies.

(1) $\sqrt{x + \sqrt{x + \sqrt{y} + z}}$ 

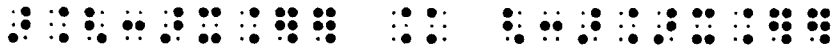
(the square root of the sum of three terms; the first term is x, the second term is the square root of x plus y; the third term is z)

(2) $\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$

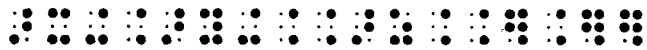


(the cube root of the sum of x squared, the cube root of x squared plus y squared, and y squared)

(3) $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$



(the square root of the cube root of x equals the cube root of the square root of x)

(4) $\sqrt{x + \sqrt{y + \sqrt{z}}}$ 

(a nest of three radicals; the outer radical contains x plus the inner radicals, the first inner radical contains y plus the second inner radical, and the second inner radical contains z)

RULE XVI—SHAPES

Shape Indicator 

Interior Shape-Modification Indicator 

Structural Shape-Modification Indicator 

Filled-in Shape Indicator 

Shaded Shape Indicator 

Termination Indicator 

Basic Shapes

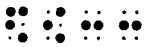
Angle  

Arc

Concave upward  

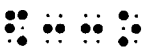
Concave downward 

Arrow

Left-pointing 

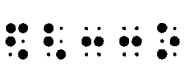
Right-pointing

Contracted 

Uncontracted  

Down-pointing  

Up-pointing



Circle

Diamond

Ellipse (oval)

Hexagon

Irregular

Regular

Intersecting Lines

Is Parallel To

Is Not Parallel To

Is Perpendicular To

Is Not Perpendicular To

Parallelogram

Pentagon

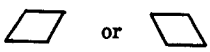
Irregular

Regular

Quadrilateral

Rectangle

Rhombus



Square

Star

Trapezoid

Triangle

Inverted 

Regular (equilateral) 

Shapes with Interior Modification

Angle

Angle with interior arc

Angle with interior clockwise arrow 

Angle with interior counterclockwise arrow 

Circle

Circle with interior arrow pointing right

Circle with interior arrow pointing left 

Circle with interior arrow pointing right over interior arrow pointing left 

Circle with interior arrow pointing left over interior arrow pointing right 

Circle with interior arrow pointing up 

Circle with interior arrow pointing down 

Circle with interior arrow pointing up followed by interior arrow pointing down 

Circle with interior arrow pointing down followed by interior arrow pointing up 

Circle with interior cross 

Circle with interior dot 

Circle with interior minus sign 

Circle with interior plus sign 

Square

Square with interior diagonals 

Square with interior dot 

Square with interior horizontal bar 

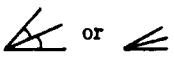
Square with interior vertical bar 

Square with interior northwest-southeast diagonal 

Square with interior southwest-northeast diagonal 

Shapes with Structural Modification

Angle

Adjacent angles 

Alternate exterior angles 

Alternate interior angles 

Complementary angles 

Corresponding angles 

Exterior angles 

Interior angles 

Obtuse angle 

Right angle 

Straight angle

Supplementary angles 

Verticle angles 

Triangle

Acute triangle 

Isosceles triangle 

Obtuse triangle 

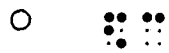
Right triangle

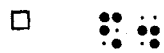
Scalene triangle

§106. **Basic Shapes:** A shape is a sign which is in general a miniature picture or diagram of the object which the sign represents.

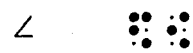
A shape is represented by using a letter, numeral, or even a configuration of dots which is suggestive of the shape. The shape indicator must precede the shape symbol. A symbol of shape must be used only for the representation of the corresponding sign of shape; it must never be used to represent the word or phrase which is the name of such a sign of shape.

(1) 

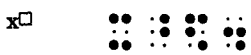
(2) 

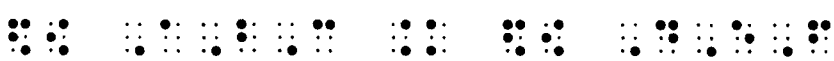
(3) 

(4) 

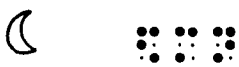
(5) 

(6) 

(7) 

(8) ∠ ABC = ∠ DEF 

§107. **Other Shapes:** Signs of shape which do not appear in the list of Basic Shapes must be represented by the use of one or more letters suggestive of the name of the shape being represented. Care must be exercised not to use an alphabetic symbol to which a meaning is already assigned in the above list. In addition, the transcriber must supply a note of explanation to the reader concerning the name of such a sign of shape and must supply a drawing of the shape if possible. If a combination of letters selected to represent a sign of shape is contractible, the contraction must not be used. The shape indicator must precede a shape symbol constructed in this way.

(1) 

(in ink print, the drawing of a moon)

(2) 

(in ink print, the drawing of a church)

§108. **Filled-In and Shaded Shapes:** Any of the closed shapes in the above list, if they are filled in or shaded, must be represented as such by using  (dots 4-5-6) or  (dots 4-6), respectively, preceding the shape symbol. The shape indicator must, in turn, precede whichever indicator has been used.

(1) 

(filled-in ellipse)

(2) 

(shaded ellipse)

§109. Polygons: The list of Basic Shapes contains the shapes for regular polygons up to six sides. Any regular polygon with more than six sides must be represented in the manner suggested, that is, by using the numeral which specifies the number of sides. An irregular polygon, that is, one which has at least two unequal sides, two unequal angles, or both, must not be represented in this way. It must be represented as specified in §107.

(1) 

(regular octagon)

(2) 

(regular polygon of 12 sides)

(3) 

(filled-in regular octagon)

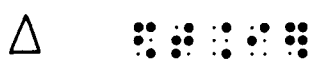
(4) 

(an irregular octagon; a transcriber's note with a drawing is required)

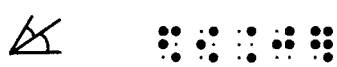
§110. Shape with Structural Modification: When a sign, which is a special case of a more general situation, is used, (for example, *right angle* is a special case of *angle*), or when two or more signs of shape are combined to form a composite sign with a more detailed structure, (for example, two *angles* are combined to form *adjacent angles*), the shape which is formed in either of these ways is called a *shape with structural modification.*

The modification is indicated by a letter or combination of letters suggestive of the nature of the modification. The symbol used for indicating the modification must be preceded by the structural shape-modification indicator and followed by the termination indicator. This combination must directly follow the symbol of basic shape which is being modified.

Shapes with structural modification not shown in the list of Shapes with Structural Modification must be transcribed in accordance with the principle suggested by those shape symbols. The transcriber must supply a note of explanation to the reader concerning the name of the structural modification and must supply a drawing if possible. If a combination of alphabetic symbols selected for a structural modifier constitutes a contractible combination, the contraction must not be used.

(1) 

(an isosceles triangle; without modification the shape symbol signifies *triangle*)

(2) 

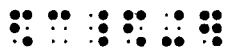
(adjacent angles; without modification the shape symbol signifies *angle*)

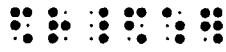
§111. Shape with Interior Modification:

a. When a letter, operation sign, or other sign is placed inside the basic sign of shape, the shape which is formed in this way is called a *shape with interior modification.*

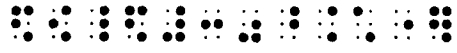
The modification is indicated by using the symbol which corresponds to the modifying sign. This symbol must be preceded by the interior shape-modification indicator and followed by the termination indicator. This combination must directly follow the symbol of basic shape which is being modified.

(1) 

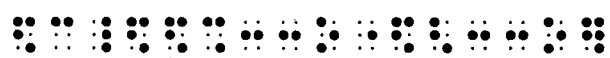
(2) 


(3) 


(4) 



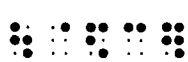
b. If two or more interior modifiers, arranged horizontally, occur inside the same basic sign of shape, the corresponding symbols must be separated by the multipurpose indicator, but the interior shape-modification indicator must be used only once, before the first modifying symbol. The entire combination must directly follow the basic symbol of shape which is being modified.

(1) 


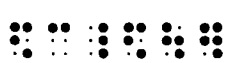
c. If two or more interior modifiers, arranged vertically, occur inside the same basic sign of shape, the corresponding symbols must be transcribed successively, without intervening spaces or indicators, beginning with the symbol which corresponds to the uppermost sign and proceeding in descending order. The first modifying symbol must be preceded by the interior shape-modification indicator, and the entire combination must directly follow the basic symbol of shape which is being modified. None of the interior signs may be regarded as a modifier of any of the others, and the technique for representing modified expressions does not apply.

(1) 


§112. Shape modified by Superposition: When a sign is superposed upon a sign of shape, the shape which is formed in this way is called a *shape modified by superposition.* Superposition may be distinguished from interior modification by noting that in superposition one of the signs extends beyond the boundary of the other. Whereas, in the case of interior modification, one of the signs is confined within the boundary of the other (see §93).

(1) 


(a vertical bar extending beyond the boundary of a circle)

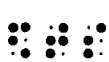
(2) 


(a circle with a vertical bar through the center extending as far as the boundary of the circle)

§113. Drawn-In Shapes: It is often better for the reader to have shapes drawn in than to have them represented by the elaborate braille constructions specified in this rule. However, it is not possible to formulate specific rules concerning which form should be used and, therefore, the decision is left to the experience and judgment of the transcriber.

§114. Plural of a Sign of Shape: The plural or the possessive of a sign of shape is sometimes indicated by placing the letter "s" on the inside of the sign of shape. When this form is employed, the braille transcription is effected simply by placing the lower-case letter "s" after the shape symbol (see §39).

(1) 



(in ink print the "s" is inside the triangle shape)

§115. Spacing with Symbols of Shape:

a. When a sign of shape is followed by its identification such as a letter, sequence of letters, or numeral, there must be a space between the shape symbol and its identification. In principle, the spacing rule which covers symbols of shape which are identified are the same as those which apply to function names and their abbreviations.

(1) $\angle 1$

(2) $\triangle$ ABC

(3) $\circ$ R

(4) In $\triangle$ ABC, $\angle A = 90°$.

(5) $\overset{s}{\triangle}$ UVW and XYZ

(6) $\llcorner A$

(7) $\measuredangle$ ABC

(8) $\angle x + \angle y$

(9) $\angle 1 + 2\angle 3$

(10) $\frac{\triangle ABC}{\triangle EFG}$

(11) m $\angle$ABC

(12) $\angle 90° + \angle 120°$

b. Shape symbols which represent omission must be spaced in accordance with the omitted item which they represent.

(1) $\square\%$

(the square represents an omitted numeral)

(2) $\$\triangle$

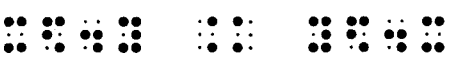
(the triangle represents an omitted numeral)

(3) $6\frac{4}{12} = 6\frac{\triangle}{3}$

(the triangle represents an omitted numeral)

(4) 1 day = 24 $\lozenge$

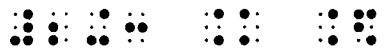
(the diamond represents an omitted word or abbreviation)

(5) $x \square y = y \square x$ 

(the square represents an omitted sign of operation)

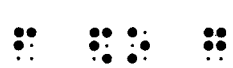
(6) $2 + 4 \triangle 7$ 

(the triangle represents an omitted comparison sign)

(7) $2 + 3 = \nabla$ 

(the inverted triangle represents an omitted numeral)

c. Symbols of shape which are either comparison symbols or operation symbols must be spaced accordingly.

(1) $f \rightarrow g$ 

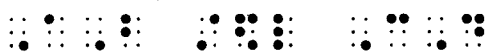
(the arrow is a comparison symbol)

(2) $\lim_{x \rightarrow \infty} f(x)$ 

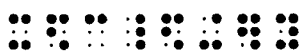
(the arrow is a comparison symbol)

(3) $AB \perp CD$ 

("is perpendicular to" is a comparison symbol)

(4) $AB \nparallel CD$ 

("is not parallel to" is a comparison symbol)

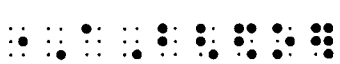
(5) $x \oplus y$ 

(the circle with interior plus symbol is a symbol of operation)

(6) $x \blacksquare y$ 

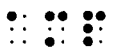
(the filled-in square symbol is a symbol of operation)

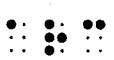
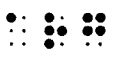
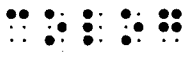
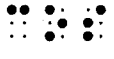
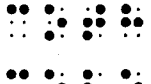
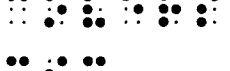
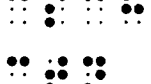
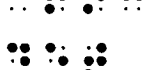
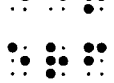
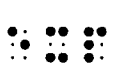
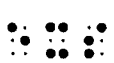
d. In any case, a symbol of shape must be unspaced from any braille indicator which applies to it.

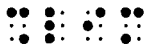
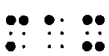
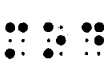
(1) $\overrightarrow{AB}$ 

(2) $1101_{\diamond} + 1000_{\diamond}$ 

RULE XVII—FUNCTION NAMES AND THEIR ABBREVIATIONS

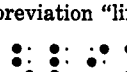
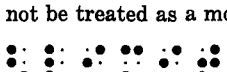
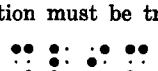
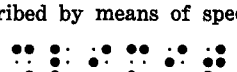
Abbreviation	Function Name	Braille Equivalent
amp	**amplitude**	
antilog	**antilogarithm**	

Abbreviation	Function Name	Braille Equivalent
arc	arc	
arg	argument	
colog	cologarithm	
cos	cosine	
cosh	hyperbolic cosine	
cot	cotangent	
coth	hyperbolic cotangent	
covers	coversine	
csc	cosecant	
csch	hyperbolic cosecant	
ctn	cotangent	
ctnh	hyperbolic cotangent	
det	determinant	
erf	error function	
exp	exponential	
exsec	exsecant	
grad	gradient	
hav	haversine	
im	imaginary part	
inf	infimum	
lim	limit	
$\overline{\lim}$	upper limit	

Abbreviation	Function Name	Braille Equivalent
lim	lower limit	
ln	natural logarithm	
log	logarithm	
max	maximum	
min	minimum	
mod	modulo	
re	real part	
sec	secant	
sech	hyperbolic secant	
sin	sine	
sinh	hyperbolic sine	
sup	supremum	
tan	tangent	
tanh	hyperbolic tangent	
vers	versine	

§116. Contractions in Function Names and Their Abbreviations: See §55b and §56.

§117. Numeric Subscripts with Function Names and Their Abbreviations: See §77.

§118. Modifiers with Function Names and Their Abbreviations: The bar which occurs over or under the function name "limit" or its abbreviation "lim" must not be treated as a modifier; the combination must be transcribed by means of special symbols for *upper limit*  or  or *lower limit*  or  . Other modifiers, however, must be transcribed in accordance with the techniques for the representation of modified expressions.

(1) $\overline{\lim}_{n \to \infty} f_n(x)$

(2) $\underline{\lim}_{n \to \infty} f_n(x)$

(3) $\lim_{x \to 0} f(x)$

§119. **Spacing with Function Names and Their Abbreviations:**

a. A space must be left after an unmodified function name or its abbreviation. If the function name or its abbreviation carries a superscript, subscript, or modifier, the space must follow the superscript, subscript, or termination of modifier.

(1) $\sin x$

(2) $\cos^2 x$

(3) $e^{\sin x}$

(4) arc AOB

(5) $\log_a x$

(6) $\lim_{x \to 0} f(x)$

b. If two or more consecutive function names or their abbreviations occur, the space between them may either be omitted or included in accordance with the ink print copy. When there is doubt concerning the presence of a space in ink print between the function names or their abbreviations, a space should be left in the transcription.

(1) arcsin x

(no space in ink print between arc and sin)

(2) arc sin x

(space between arc and sin clearly shown in ink print)

c. The expression which follows or precedes the function name or its abbreviation must be spaced in accordance with the other spacing rules of this Code.

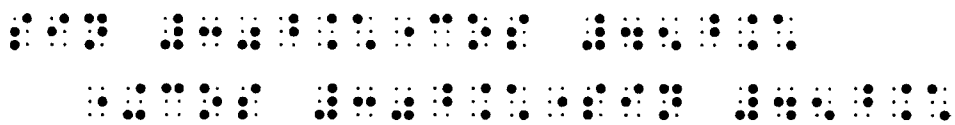
(1) $\sin x + y$

(in ink print, there is a space on both sides of the plus sign)

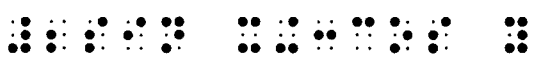
(2) $\sin \pi/3$

(in ink print, there is no space on either side of the diagonal line)

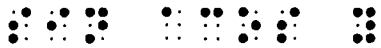
(3) sin 30° cos 45° + cos 30° sin 45°



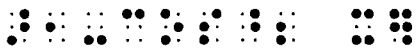
(in ink print, cos 45° and cos 30° are preceded and followed by spaces)

(4) 2sin x + 3cos y 

(in ink print, there is no space after the 2 and the 3, and there is a space on both sides of the plus sign)

(5) sin a cos y 

(in ink print, there is a space on both sides of the a and the y)

(6) $\sqrt{1 - \cos^2 x}$ 

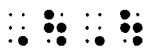
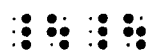
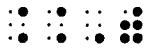
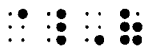
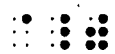
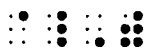
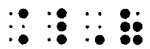
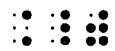
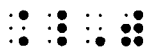
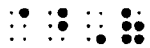
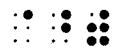
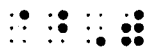
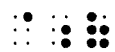
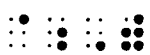
(in ink print, there is a space on both sides of the minus sign and the x)

(7) $\frac{1}{\cos} - \cos = \tan \cdot \sin$ 

(8) 1/cos — cos = tan · sin

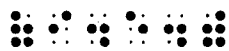
RULE XVIII—SIGNS AND SYMBOLS OF GROUPING

		Normal	Enlarged
Parentheses (round brackets)			
Left	(		
Right	)		
Brackets (square brackets)			
Left	[		
Right	]		
Boldface Left	[		
Boldface Right	]		

		Normal	Enlarged
Braces (curly brackets)			
Left	{		
Right	}		
Vertical Bar			
Single	|		
Double	||		
Boldface Single	|		
Boldface Double	||		
Angle Brackets (angular parentheses)			
Left	⟨		
Right	⟩		
Barred Brackets			
Left	[		
Right	]		
Barred Braces			
Left	{		
Right	}		
Half Brackets			
Upper Left	⌜ or ⌜		
Upper Right	⌝ or ⌝		
Lower Left	⌞ or ⌞		
Lower Right	⌟ or ⌟		
Transcriber's Grouping Symbols			
Left			
Right			

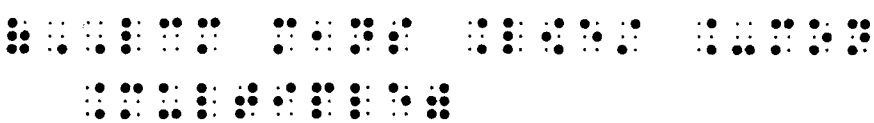
§120. Symbols of Grouping:

a. The grouping symbols listed above must be used everywhere in the transcription, whether for literary or mathematical purposes. The grouping symbols of English Braille must never be used except to enclose literary material on title pages. If a grouping sign occurs which is not listed above, then in the spirit of the Code, the transcriber must devise a symbol whose first component is ⠸⠣ for the opening grouping symbol and must use the same device but with last component ⠸⠜ for the closing symbol.

(1) (i.e.) 

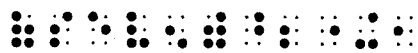
(2) (s.a.s. = s.a.s.) 

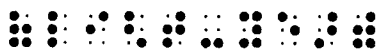
(3) (LCM means *lowest common multiple*)



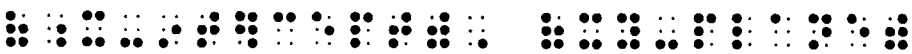
(4) {Wed., Thurs., Fri.} 

(5) {Mary, Sally, Jean} 

(6) (seven)^2 + 1 

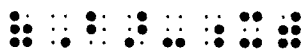
(7) (light-year) 

(8) (x-intercept), (xy-plane)



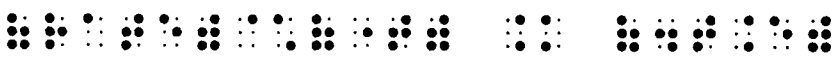
(9) ("Two" is not the same as two.)

(10) (5-inch stick) 

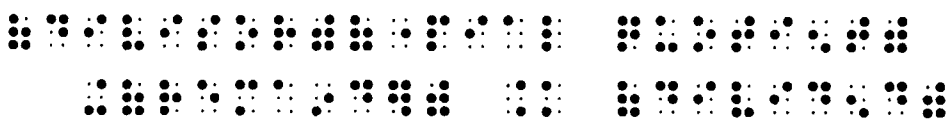
(11) (Bar-x) 

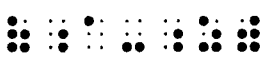
(12) (1-to-1) 

(13) (rate) × (time) = (distance)

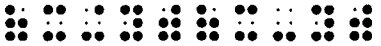


(14) (divisor) (partial quotient) + (remainder) = (dividend)

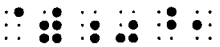


(15) (a-z) 

(16) (VI-IX) 

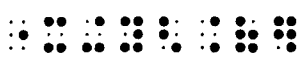
(17) $(x + y)(x - y)$ 

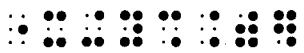
b. Although signs of grouping most commonly occur in pairs, this is not always so. If an opening grouping sign occurs without being followed later by the corresponding closing sign, or if the closing sign occurs without having been preceded by the corresponding opening sign, this situation must be preserved in the transcription.

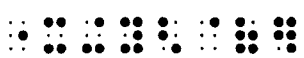
(1) $]_0^1$ 

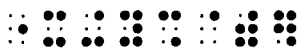
(closing bracket with 0 as subscript and 1 as superscript)

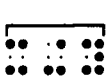
§121. Horizontal Grouping Signs: When a horizontal grouping sign occurs over or under a mathematical expression, it must be regarded as a modifier. It is recommended that the horizontal grouping symbols be drawn. However, when they are to be represented in braille, the modified expression must be transcribed according to **§86a.** The left grouping symbol must be used when the modifier is *directly over* and the right grouping symbol when the modifier is *directly under.*

(1) $\overbrace{x + y}$ 

(2) $\underbrace{x + y}$ 

(3) $\overline{x + y}$ 

(4) $\underline{x + y}$ 

(5) $\overline{x + y}$ 

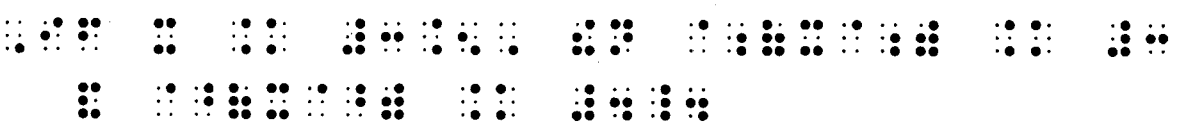
(6) $\overbrace{x + y}$ 

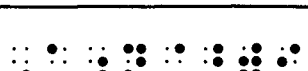
§122. Boldface Brackets: Boldface brackets are often used to designate the *integer function.*

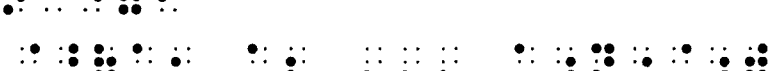
(1) $\mathbf{[x]}$ 

§123. Half-Brackets: The upper half-brackets (left and right) are commonly used to represent the *ceiling function.* The lower half-brackets (left and right) are commonly used to represent the *floor function.* These signs are also used for miscellaneous purposes in many fields of mathematics and science.

(1) If $x = 3.5$, then $\lfloor x \rfloor = 3$ and $\lceil x \rceil = 4$.

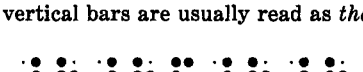


(2) $A_{n\,1i}$ 

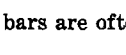
(3) $\lceil a_1\ a_2\ \ldots\ a_n \rfloor$ 

§124. Vertical Bars:

a. Double boldface vertical bars are usually read as *the norm of.*

(1) $\| f \|$ 

b. Single vertical bars are often read as *the absolute value of,* but are used for other purposes.

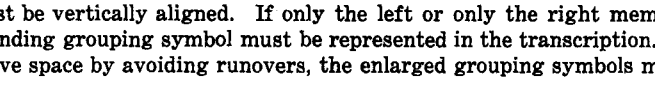
(1) $|x|$ 

(2) $\Big|_0^1$ 

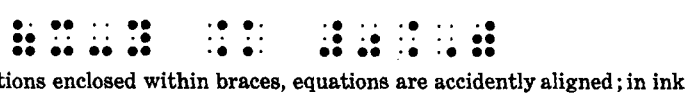
(3) $\Big|_{x=0}$ 

§125. Transcriber's Grouping Symbols: The regular transcriber's grouping symbols in the above list must be used to enclose any transcriber's note which has been inserted into the text (see §186b). These must not be used to enclose a list of transcriber's notes which appears at the beginning of a braille volume. The same rules which govern punctuation and contraction of expressions containing grouping symbols also govern transcriber's notes. For use of enlarged transcriber's grouping symbols see §184b.

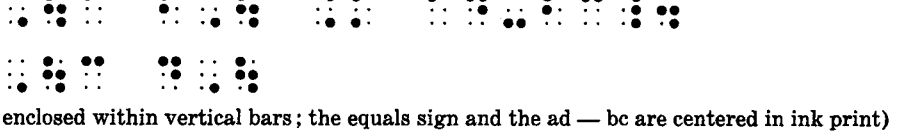
§126. Use of Enlarged Grouping Symbols: When a system of mathematical expressions is arranged on two or more lines of ink print, and a sign of grouping is used to unify the system, the corresponding grouping symbol in the transcription must be indicated as enlarged by the use of dot 6 to indicate the enlargement. Among such systems of mathematical expressions are: systems of equations, determinants, and matrices. Each braille line which contains any part of the transcription of such a system must also contain the enlarged grouping symbol and these must be vertically aligned. If only the left or only the right member of a pair of grouping signs is present in ink print, only the corresponding grouping symbol must be represented in the transcription. However, when it is advisable for any reason to do so, for example, to save space by avoiding runovers, the enlarged grouping symbols may be drawn.

(1) $\left\{\begin{matrix} x + y = 2 \\ x - y = 0 \end{matrix}\right\}.$ 

(a two-line system of equations enclosed within braces, equations are accidently aligned; in ink print, period is centered)

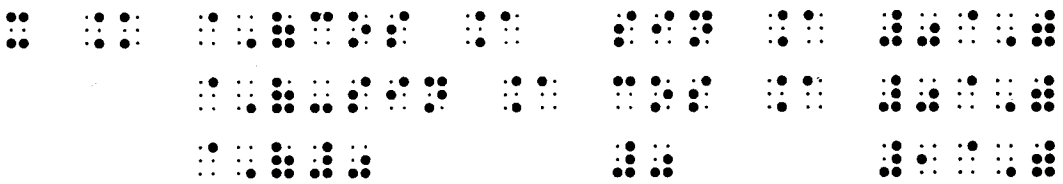
(2) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$ 

(a two-by-two determinant enclosed within vertical bars; the equals sign and the ad — bc are centered in ink print)

(3) $y = \begin{cases} x, \text{ if } x \leqq 0 \\ 0, \text{ if } x > 0. \end{cases}$ 

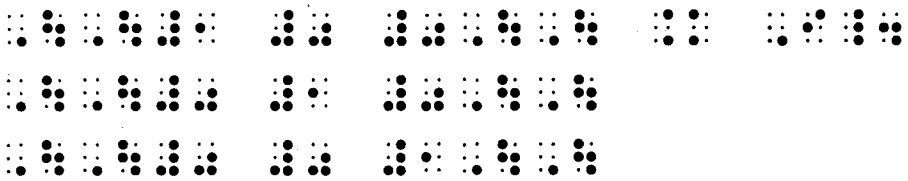
(a two-line system unified on the left by a left bracket)

(4) $x = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$



(a three-by-three matrix enclosed within large brackets)

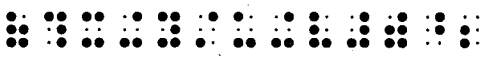
(5) $\left\| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right\| = I.$



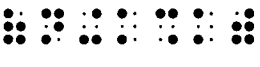
(a three-by-three matrix enclosed within double vertical bars)

§127. Non-Use of Enlarged Grouping Symbols: Signs of grouping must not be indicated as enlarged in the transcription when the corresponding signs in ink print are made large such as for the purpose of covering a fraction, binomial coefficient or other material occupying a large amount of vertical space. No signs, except grouping signs, may ever be shown as being enlarged.

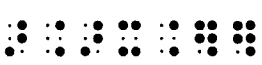
(1) $\left(\frac{x+y}{u+v}\right)^2$



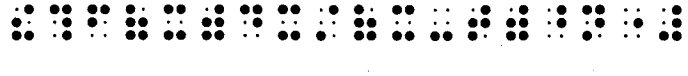
(2) $\binom{n+k}{k}$



(3) $\sqrt{\sqrt{x}}$



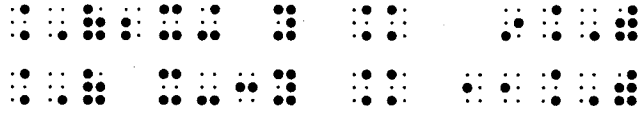
(4) $\int \frac{f(x)\,dx}{(x-t)^n}$



§128. Spacing with Symbols of Grouping:

a. Spaces may be required to be left after an opening enlarged grouping symbol or before a closing enlarged grouping symbol to preserve the vertical alignment of such symbols required in §126.

(1) $\left\{ \begin{matrix} 2x + y = 9 \\ x - 3y = 11 \end{matrix} \right\}$



(a unified system of two equations in which vertical alignment is required)

b. A space must be left between an opening and closing grouping symbol when there is a blank, not representing omission, between the corresponding signs in ink print.

(1) { } 

(the empty set)

RULE XIX—SIGNS AND SYMBOLS OF OPERATION

Ampersand (and, logical product) &

Asterisk *

Back Slash (divides, is a factor of) \

Circle with Interior Dot ⊙

Circle with Interior Plus ⊕

Circle with Interior Minus ⊖

Dagger

Single †

Double ‡

Division (divided by) ÷

Dot (and) ·

Fraction Line (over)

Diagonal /

Simple —

Diagonal Complex /

Complex —

Hypercomplex —

Hollow Dot °

Intersection (cap) ∩

Logical Product (and, meet) ∧

Logical Sum (join, or) ∨

Minus

Regular —

Boldface —

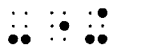
Minus Followed by Plus

Boldface Minus Followed by Boldface Plus —+

Boldface Minus Followed by Regular Plus —+

Regular Minus Followed by Regular Plus —+

Regular Minus Followed by Boldface Plus —+



Minus or Plus ∓

Minus with Dot over (proper difference) ∸

Multiplication (times)

Cross (cartesian product) ×

Dot ·

Number Sign; Crosshatch; Tic-tac-toe; Pounds (weight) #

Paragraph Mark ¶

Plus

Regular +

Boldface +

Plus Followed by Minus

Boldface Plus Followed by Boldface Minus +—

Boldface Plus Followed by Regular Minus +—

Regular Plus Followed by Regular Minus + —

Regular Plus Followed by Boldface Minus + —

Plus or Minus ±

Section Mark §

Slash (per, over, divided by) /

Square

Filled-In Square ■

Hollow Square □

Star ☆

Tilde

Extended ≈

Simple ~

Union (cup) ∪

Vertical Bar (is a factor, divides) |

§129. Ampersand: When the ampersand is used in literary context, and without reference to the specific nature of the sign itself, it should not be regarded as a sign of operation, and, accordingly, the rules of English Braille apply. Otherwise, the symbol in the above list must be used.

(1) AT&T

(literary context)

(2) The & often denotes logical conjunction.

(specific attention is called to the nature of the sign)

(3) A & B

(mathematical context)

§130. Asterisk, Dagger, Double Dagger, Number Sign, Paragraph Mark, Section Mark, Star: The asterisk, dagger, double dagger, number sign, paragraph mark, section mark, and star must be represented by the symbols provided for them in this Code; the English Braille symbols must be avoided even when the corresponding signs are used for purposes of reference.

(1) f * g

(2) 3 * 4

(3) x*

(4) x # y

(5) 2#3

(6) R#

(7) A ¶ B

(8) A § B

(9) A ☆ B

§131. Fraction Lines: For a complete discussion of the rules governing fractions see **Rule XII.**

§132. Intersection, Union: When the intersection sign or the union sign is modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see §147).

These signs are frequently modified below, and are consequently printed wide enough to accommodate the modifier. The variable width of these signs must be ignored in the transcription. Superscripts or subscripts which are sometimes attached to these signs must be treated in the usual manner for handling superscripts and subscripts.

(1) $A \cap B$

(2) $\bigcap_{\alpha \in A} x_\alpha$

(3) $A \cup B$

§133. Logical Product, Logical Sum: When the signs for *logical product* or *logical sum* are modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see §147).

(1) $x \wedge y$

(2) $x \vee y$

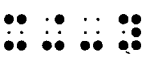
§134. Minus Followed by Plus, Plus Followed by Minus, Minus or Plus, Plus or Minus: When the signs for *plus* and *minus* are combined either vertically or horizontally, the combination must be regarded as a single sign of operation. Its components must not be divided between braille lines in the transcription.

(1) $+2 - +3$

(2) $-3 + -5$

(3) $x \mp y$

(4) $x \pm y$



§135. Multiplication: The two common multiplication signs, *cross* and *dot*, must not be used interchangeably in the transcription. The cross is sometimes modified below.

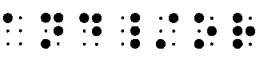
(1) 3×10

(2) $x \cdot y$

(3) $\underset{\alpha \in A}{\times} A_\alpha$

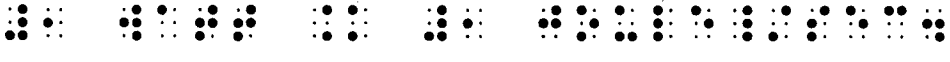
§136. Slash: The slash must be represented by the symbol provided for it in this Code. The English Braille symbol must not be used.

(1) and/or



(2) The rise/run ratio is 3.

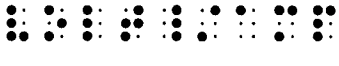
(3) 1 watt = 1 joule/sec.



(4) c/o

(abbreviation meaning "care of")

(5) volt/amp



(6) 60 mi/hr

(7) 7/4/76

(a date)

§137. Tilde: This sign of operation is used predominantly in logic with the meaning of *not.*

(1) $\sim p$

(2) $\sim p \vee \sim q \vee \sim r$

(3) $\sim\sim T \vee R$

§138 Spacing with Symbols of Operation:

a. A space must be left on either side of an operation symbol under any of the circumstances listed below.

i. Between a comparison symbol and an operation symbol.

(1) $x = -y$

ii. After a function name or its abbreviation and before a symbol of operation.

(1) $\sin -x$

iii. Between an ellipsis or dash and a symbol of operation.

(1) $1 + 2 + \ldots + n$

(2) $10 - \text{——} = 8$

iv. Between an abbreviation and a symbol of operation other than the fraction line or slash.

(1) 1 yd + 2 yd = 3 yd

(in ink print, there is a space on both sides of the plus sign)

v. Where required according to **Rule XXII.**

(1) |||| + ||||

b. A space must not be left on either side of a symbol of operation in any other situation.

(1) a\b

(2) $x \oplus y$

(3) $12 \div 3$

(4) $f \circ g$

(5) $\sin x - \sin y$

(in ink print, there is a space on both sides of the minus sign)

(6) $x \square y$

(7) $\square + \triangle$

(8) rate $\times$ time

(9) miles/hour

(10) quotient $\times$ divisor $+$ remainder $=$ dividend

(11) $3 \times \text{seven}^2 + 4 \times \text{seven}^1 + 5 \times \text{seven}^0 = 345_{\text{seven}}$

(12) $(2n + 3) \mid 3$

(13) $3\ \text{ft}^2 + 3\ \text{ft}^2 = 6\ \text{ft}^2$

RULE XX—SIGNS AND SYMBOLS OF COMPARISON

Simple Comparison Signs

Arc

Concave upward ⁀

Concave downward ‿

Arrow

Left-pointing ←

Right-pointing

Contracted →

Uncontracted →

Down-pointing ↓

Up-pointing ↑

Two-way

Horizontal ↔

Vertical ↕

Equals (is equal to)

Normal =

Boldface =

Greater Than (is greater than)

Normal >

With curved sides ≻

Identity (is congruent to; is identical to) ≡

Inclusion (is contained in; is a subset of) ⊂

Less Than (is less than)

Normal <

With curved sides ≺

Membership (is an element of; belongs to) ϵ or ∈ or ∈

Parallel To (is parallel to) ∥

Perpendicular To (is perpendicular to) ⊥

Proportion (as) ::

Ratio (is to) :

Relation (is related to) R

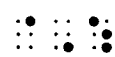
Reverse Inclusion (contains; in logic, implies) ⊃

Reverse Membership (contains the element) ∍ or ∋ or ∋

Tilde

Simple (is related to; is similar to) ~

Extended (is related to) ≈



Variation (varies as) ∝

Vertical Bar (such that) |

Modified Comparison Signs

Equals Sign

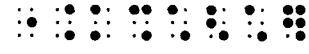
Caret over ≙

Caret under (is projective to, projective correspondence) ≚

Degree sign over (is equal in degrees to) ≗

Dot over (is approximately equal to) ≐

Dot over and dot under ≑



Equilateral triangle over ≜

Inverted caret over ≛

Left-pointing caret over ≜

Question mark over ≟

Right-pointing caret over ≜

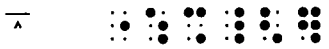
Two dots over and two dots under ≒

Vertical bar over ≜

Horizontal Bar

Caret over ≜

Caret under (is perspective to, perspective correspondence)



Dot under

Simple Tilde, Dot Under

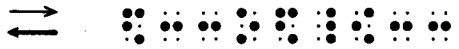
Comparison Signs Compounded Vertically

Arrow Combinations

Right-pointing over left-pointing

Right-pointing with upper barb over left-pointing with lower barb

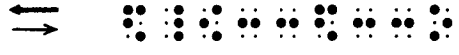
Right-pointing over boldface left-pointing



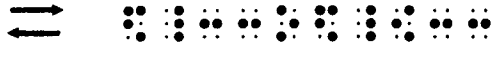
Left-pointing over boldface right-pointing

Boldface right-pointing over left-pointing

Boldface left-pointing over right-pointing



Boldface right-pointing over boldface left-pointing



Boldface left-pointing over boldface right-pointing

Long right-pointing over short left-pointing

Short right-pointing over long left-pointing

Equivalence (is equivalent to)

Greater Than

Bar over greater than (is equal to or greater than) $\overline{>}$ or $\geqslant$

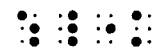
Bar under greater than (is greater than or equal to) $\underline{>}$ or $\geqslant$

Equals sign over greater than (is equal to or greater than) $\overset{=}{>}$ or $\eqslantgtr$

Equals sign under greater than (is greater than or equal to) $\underset{=}{>}$ or $\eqslantgtr$

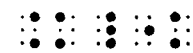
Inclusion

Bar over inclusion (is a subset of) $\overline{\subset}$



Bar under inclusion (is a subset of) $\underline{\subset}$

Equals sign over inclusion (is a subset of) $\overline{\underline{=}}\subset$

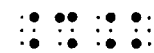


Equals sign under inclusion (is a subset of) $\underline{\overline{=}}\subset$

Intersection (cap)

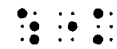
Bar under intersection $\underline{\cap}$

Equals sign under intersection $\underline{\overline{=}}\cap$



Less Than

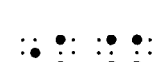
Bar over less than (is equal to or less than) $\overline{\leq}$ or $\overline{<}$



Bar under less than (is less than or equal to) $\underline{\leq}$ or $\underline{<}$

Equals sign over less than (is equal to or less than) $\overline{\underline{=}}<$ or $\overline{\overline{<}}$

Equals sign under less than (is less than or equal to) $\underline{\overline{=}}<$ or $\underline{\overline{<}}$



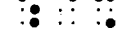
Logical Product (meet)

Bar over logical product $\overline{\wedge}$

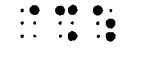
Bar over and bar under logical product $\overline{\underline{\wedge}}$

Bar over and equals sign under logical product $\overline{\underline{\wedge}}$

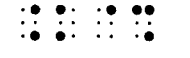
Bar under logical product $\underline{\wedge}$



Equals sign over logical product $\overline{\underline{=}}\wedge$



Equals sign over and bar under logical product $\overline{\underline{=}}\underline{\wedge}$



Equals sign over and equals sign under logical product $\overline{\underline{=}}\underline{\overline{=}}\wedge$

Equals sign under logical product $\underline{\overline{=}}\wedge$

Logical Sum (join)

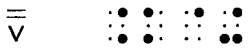
Bar over logical sum

Bar over and bar under logical sum

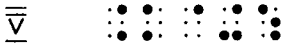
Bar over and equals sign under logical sum

Bar under logical sum

Equals sign over logical sum



Equals sign over and bar under logical sum



Equals sign over and equals sign under logical sum

Equals sign under logical sum

Reverse Inclusion

Bar over reverse inclusion

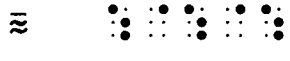
Bar under reverse inclusion

Equals sign over reverse inclusion

Equals sign under reverse inclusion

Tilde (is related to)

Bar over double tilde



Bar over single tilde

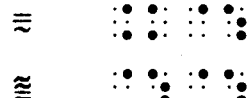
Bar under double tilde

Bar under single tilde

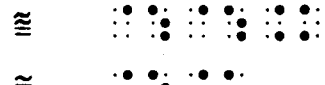
Double tilde

Equals sign over double tilde

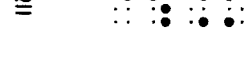
Equals sign over single tilde



Equals sign under double tilde



Equals sign under single tilde



Union (cup)

Bar under union ⊍

Equals sign under union ⊍

Comparison Signs Compounded Horizontally

Arrow Combinations

Up-pointing followed by down-pointing ↑↓

Down-pointing followed by up-pointing ↓↑

Up-pointing followed by boldface down-pointing ↑↓

Down-pointing followed by boldface up-pointing ↓↑

Boldface up-pointing followed by down-pointing ↑↓

Boldface down-pointing followed by up-pointing ↓↑

Boldface up-pointing followed by boldface down-pointing ↑↓

Boldface down-pointing followed by boldface up-pointing ↓↑

Greater Than

Followed by less than > <

Followed by equals sign followed by less than > = <

Less Than

Followed by greater than < >

Followed by equals sign followed by greater than < = >

Comparison Signs Compounded by Superposition

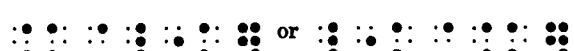
Dot

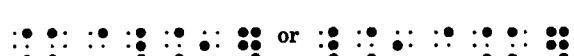
Between bars of equals sign ⊜

Within inclusion sign ⊆

Within reverse inclusion sign ⊇

Equals Sign

Through inclusion sign ⋐  or

Through reverse inclusion sign ⋑  or

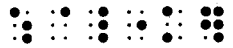
Greater Than

Nest of two with straight sides (is large compared with) ≫

Nest of two with curved sides ⪢

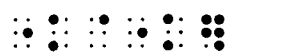
Horizontal Bar

Through inclusion sign ⊂

Through reverse inclusion sign ⊃ 

Less Than

Nest of two with straight sides (is small compared with) ≪

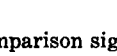
Nest of two with curved sides ⪡ 

Vertical Bar

Through shaft of right-pointing arrow 

Through shaft of left-pointing arrow

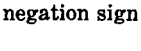
§139. **Negation:** Comparison signs may be negated by a vertical stroke or by an oblique stroke in either direction. However the negation is effected in ink print, the symbol (dots 3-4) must be placed unspaced before the comparison symbol being negated.

(1) ≠ 

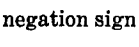
(oblique negation sign in ink print, from lower left to upper right)

(2) ⫧

(vertical negation sign in ink print)

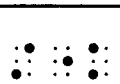
(3) ≸ 

(oblique negation sign in ink print, from upper left to lower right)

(4) ≹ 

(oblique negation sign in ink print, from lower left to upper right)

(5) $\not<$



(oblique negation sign in ink print, from upper left to lower right)

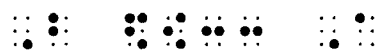
(6) $\not\geq$

(vertical negation sign in ink print)

§140. **Arrows:** A detailed discussion of the construction of arrows of many types is presented in **Rule XXI.** The arrows in the list of simple comparison signs are those which occur with the greatest frequency.

If a right-pointing arrow has a full barb and a single shaft of ordinary length, is in regular type, and occurs by itself, it must be represented in its contracted form. If such an arrow is in nonregular type, is itself modified, or occurs as part of a more complex modification, it must be represented in its uncontracted form.

(1) $B \leftarrow A$



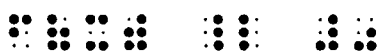
(2) $A \rightarrow B$

(3) $A \leftrightarrow B$

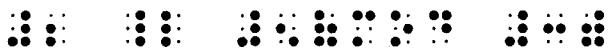
(4) $X \xrightarrow{f \circ g} Y$

§141. **Identity:** This sign must not be used for *is congruent to* in geometry if another sign is employed for this purface in ink print.

(1) $f(x) \equiv 0$



(2) $2 \equiv 5 \pmod 3$



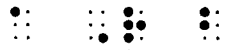
§142. **Membership:** This sign must not be mistaken for the Greek lower-case epsilon, even though it is sometimes referred to by that very name. This sign is generally used when speaking about sets and the elements of which they are composed. When the Greek lower-case epsilon is used in the same textbook, the publisher usually makes a sufficient distinction between the two signs to prevent this confusion.

(1) $x \in A$

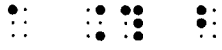
(x is an element of A)

§143. **Relation:** When a letter or other sign is used between two expressions to show that they are related, the letter or sign used in this way must be regarded as a comparison sign. As such, it is subject to all the rules governing comparison signs and symbols. The letter R is frequently used in this situation.

(1) a R b



(2) $a \theta b$



§144. Tilde: When the tilde, simple or extended, occurs with a dot or caret directly over or directly under it, the combination is a modified sign of comparison (see §146). When it occurs directly over or directly under another simple comparison sign, the combination is a comparison sign compounded vertically (see §147).

(1) $x \sim y$

(2) $x \approx y$

§145. Vertical Bar: In addition to its use as a comparison sign meaning "such that," the vertical bar is used in several other ways in mathematics. It has already been listed as a sign of grouping and as a sign of operation. It is helpful to know that when the vertical bar means "such that" it is usually part of an expression within braces used for set notation, or in association with one of the quantifiers. However, it may also appear in other situations.

(1) $\{x \mid |x| < 10\}$

(2) Choose $x \mid x = y^2$.

(3) $\exists|_x \mid x = -x$

§146. Modified Comparison Signs: The modified comparison signs in the above list are constructed in accordance with the rules for the representation of modified expressions (see **Rule XIV**). Modified signs of comparison other than those in the above list must be constructed in accordance with the same principles.

§147. Comparison Signs Compounded Vertically: The transcriber must represent a vertical arrangement of simple comparison signs as an unspaced horizontal succession of the corresponding simple comparison symbols, the first symbol corresponding to the uppermost sign. The braille reader must interpret a succession of unspaced simple comparison symbols as representing the fact that the corresponding signs are arranged vertically in ink print, in descending order, the uppermost sign corresponding to the first symbol. Comparison signs compounded vertically not shown in the list must be transcribed in accordance with the above principles.

§148. Intersection, Union, Logical Product, Logical Sum: The intersection, union, logical product, and logical sum signs, when unmodified, are not simple comparison signs, but operation signs (see §132 and §133, respectively).

§149. Comparison Signs Compounded Horizontally: The transcriber must represent a horizontal succession of comparison signs by placing the multipurpose indicator between the unspaced corresponding comparison symbols. Comparison signs compounded horizontally which are not shown in the list must be transcribed in accordance with the above principle.

§150. Comparison Signs Compounded by Superposition: Comparison signs compounded by superposition in the above list are constructed in accordance with the rules for representing superposition (see §93). Comparison signs compounded by superposition other than those in the above list must be constructed in accordance with the same principles.

§151. Spacing with Symbols of Comparison: A space must be left on either side of a comparison symbol. However, a space must not be left between the comparison symbol and any punctuation symbol, grouping symbol, or indicator which applies to it.

(1) $x \frown y$

(2) $x = y$

(3) $\sum_{n=0}^{\infty} a_n$

(4) (a, b) = (c, d) if and only if a = c and b = d.

(5) $x > y$

(6) $X \subset Y$

(7) $\bigcup_{A \subset F} A$

(8) $\sum_{i<j} a_{ij}$

(9) $A \ni x$

(10) 1 : 2 :: 3 : 6

(11) a + b : b :: c + d : d

(12) $x \propto y$

(13) {all x | each $x < 6$}

(14) The unit interval = $\{x \mid 0 \leq x \leq 1\}$

(15) The symbol for *less than* is "<."

(16) (<, =, >)

(17) $\int_{x=a}^{x=b} f(x)\ dx$

RULE XXI—ARROWS

Arrow Components

Arrow Direction Indicators

Depresses nearer arrowhead by 45 degrees

Elevates nearer arrowhead by 45 degrees

Makes nearer arrowhead point up

Makes nearer arrowhead point down

Arrow Shafts

Curved (or)

Dashed

Dotted

Long double

Long single

Ordinary double

Ordinary single

Short double

Short single

Wavy

Arrow Types

Boldface

Regular (no indicator)

Arrowheads

Barbed left full

Barbed left lower

Barbed left upper	⠈	⠉ ⠩
Barbed right full	⠕	⠭
Barbed right lower	⠘	⠉ ⠭
Barbed right upper	⠐	⠩ ⠭
Blunted left full	⠫	⠿
Blunted left lower	⠇	⠉ ⠿
Blunted left upper	⠋	⠩ ⠿
Blunted right full	⠱	⠿
Blunted right lower	⠸	⠉ ⠿
Blunted right upper	⠓	⠩ ⠿
Curved left full	⠣	⠷
Curved left lower	⠁	⠉ ⠷
Curved left upper	⠈	⠩ ⠷
Curved right full	⠜	⠾
Curved right lower	⠄	⠉ ⠾
Curved right upper	⠐	⠩ ⠾
Straight left full	⠍	⠽
Straight left lower	⠃	⠉ ⠽
Straight left upper	⠅	⠩ ⠽
Straight right full	⠪	⠽
Straight right lower	⠂	⠉ ⠽
Straight right upper	⠊	⠩ ⠽

§152. Contracted Form of Right-Pointing Arrow: When a right-pointing arrow in regular type, with a single shaft of ordinary length and a full barb, occurs by itself, it must be represented in its contracted form  . If such an arrow is in non-regular type, is modified, or occurs as part of a more complex modification, it must be represented in its uncontracted form  .

§153. Arrow Components: There is a large class of signs in the form of arrows which differ from each other in several ways. In the above list, there is presented an assortment of arrow components from which such signs are constructed. The entire construction is a simple comparison symbol.

§154. Six Steps for Construction of Arrows: Arrow components must be transcribed in the following order:

i. The shape indicator.

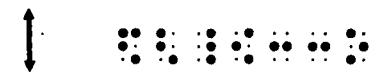
ii. The arrow direction, if it must be indicated.

iii. The arrow type, if it must be indicated.

iv. The left arrowhead, if any.

v. The arrow shaft, if required.

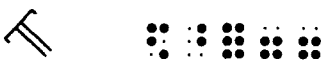
vi. The right arrowhead, if any.

(1) 

(arrow, two-way vertical, boldface, barbed arrowheads at both ends)

(2) 

(arrow, two-way horizontal, regular type, curved arrowheads at both ends)

(3) 

(spear, northwest, blunted arrowhead)

§155. Arrow Directions: It is possible to represent eight arrow directions by making proper use of the direction indicators.

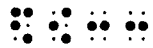
a. The two horizontal directions, *right* and *left*, require no indicator.

(1) 

(right-pointing arrow, contracted form)

(2) 

(right-pointing arrow, uncontracted form)

(3) ← 

(left-pointing arrow)

(4) ↔ 

(two-way horizontal arrow)

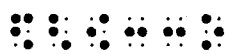
b. The two vertical directions, *up* and *down*, require the directly-over indicator or the directly-under indicator, respectively. The directly-over indicator "makes the arrowhead point up"; the directly-under indicator "makes the arrowhead point down". If a vertical arrow is printed with one arrowhead, it must be transcribed by using the appropriate symbol for a *right* arrowhead, and not a left one.

(1) ↑ 

(arrow pointing up)

(2) ↓ 

(arrow pointing down)

(3) ↕ 

(vertical two-way arrow)

c. The four oblique directions require the superscript indicator or the subscript indicator. The superscript indicator "elevates the nearer arrowhead (if there are two) by 45 degrees from the horizontal position"; the subscript indicator "depresses the nearer arrowhead (if there are two) by 45 degrees from the horizontal position".

(1) ↗ 

(arrow, northeast)

(2) ↖ 

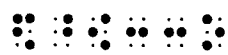
(arrow, northwest)

(3) ↘ 

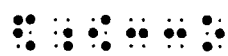
(arrow, southeast)

(4) ↙ 

(arrow, southwest)

(5) ⤡ 

(arrow, northwest-southeast)

(6) ⤢ 

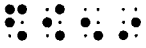
(arrow, southwest-northeast)

§156. Arrow Shafts: An arrow shaft may be curved, dashed, dotted, straight or wavy, single or double, long or short.

a. If an arrow shaft is curved, the direction of curvature is indicated by a left arrowhead or a right arrowhead. A curved arrow shaft followed by a right arrowhead represents a counterclockwise arrow; a curved arrow shaft preceded by a left arrowhead represents a clockwise arrow.

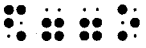
(1)  

(arrow, counterclockwise)

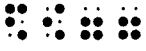
(2)  

(arrow, clockwise)

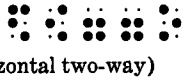
b. Most arrow shafts are single. An arrow with a double arrow shaft is sometimes called a *spear*.

(1) ⟹ 

(spear, right-pointing)

(2) ⟸ 

(spear, left-pointing)

(3) ⟺ 

(spear, horizontal two-way)

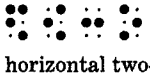
c. Where the length of an arrow shaft has significance, the length is indicated by the number of repetitions of the braille arrow shaft symbol. The list distinguishes three lengths, but other lengths may be indicated by repeating the braille arrow-shaft symbol a suitable number of times.

(1) → 

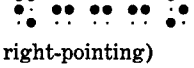
(short arrow, right-pointing)

(2) ← 

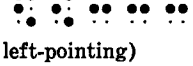
(short arrow, left-pointing)

(3) ↔ 

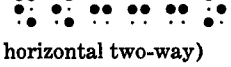
(short arrow, horizontal two-way)

(4) ⟶ 

(long arrow, right-pointing)

(5) ⟵ 

(long arrow, left-pointing)

(6) ⟷ 

(long arrow, horizontal two-way)

§157. Arrow Types: Most arrows are printed in regular type. In that case, no indicator is required. If an arrow is printed in boldface type, the boldface type indicator is required.

(1) → 

(boldface arrow, right-pointing)

(2) ← 

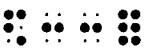
(boldface arrow, left-pointing)

(3) ↔ 

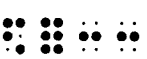
(boldface arrow, horizontal two-way)

§158. Arrowheads:

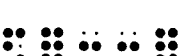
a. Most arrowheads are barbed. However, arrowheads also occur as blunted, curved, or straight. They may occur at the left end, right end, or at both ends, of the arrow shaft.

(1) ⟞ 

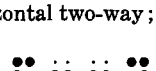
(arrow, right-pointing; blunted arrowhead)

(2) ⟝ 

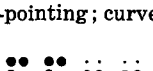
(arrow, left-pointing; blunted arrowhead)

(3) ⟞⟝ 

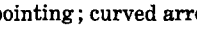
(arrow, horizontal two-way; blunted arrowheads)

(4) → 

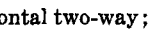
(arrow, right-pointing; curved arrowhead)

(5) ← 

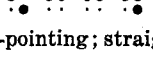
(arrow, left-pointing; curved arrowhead)

(6) ↔ 

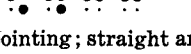
(arrow, horizontal two-way; curved arrowheads)

(7) ⟞ 

(arrow, right-pointing; straight arrowhead)

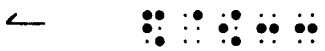
(8) ⟝ 

(arrow, left-pointing; straight arrowhead)

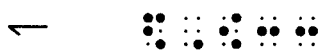
(9) ⟞⟝ 

(arrow, horizontal two-way, straight arrowheads)

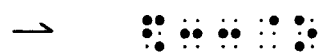
b. An arrowhead with its upper half only, or its lower half only, may also be present. Any combination of arrowheads — barbed, blunted, curved, straight, left or right, full, lower half, or upper half — may occur.

(1) 

(arrow, left upper barb only)

(2) 

(arrow, left lower barb only)

(3) 

(arrow, right upper barb only)

(4) 

(arrow, right lower barb only)

(5) 

(arrow, upper barbs only)

(6) 

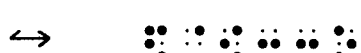
(arrow, lower barbs only)

(7) 

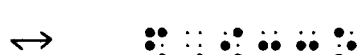
(arrow, left upper barb and right lower barb)

(8) 

(arrow, left lower barb and right upper barb)

(9) 

(arrow, left upper barb and full right barb)

(10) 

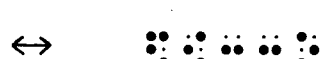
(arrow, left lower barb and full right barb)

(11) 

(arrow, full left barb and right upper barb)

(12) 

(arrow, full left barb and right lower barb)

(13) 

(arrow, full left and right barbs)

RULE XXII—MISCELLANEOUS SIGNS AND SYMBOLS

Angstrom Unit Å

At @

Caret (circumflex) ^

Cent ¢

Check Mark ✓

Crossed d đ

Crossed h ħ

Crossed Lambda ƛ

Crossed R ₹

Degree °

Del (nabla, gradient) ∇ or ▽

Ditto Mark "

Dollar $

Empty Set

Represented by Zero with Vertical or Oblique Bar Through It ϕ or ∅

Represented by Facing Braces { }

Factorial !

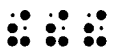
Infinity ∞

Integral

Single ∫

Double ∫∫

Triple ∫∫∫



Lower ∫

Upper ⌠

Integral with Superposed Circle ∮

Integral with Superposed Infinity ∯

Integral with Superposed Rectangle ⨏

Integral with Superposed Square ⨑

Partial Derivative (round d) ∂

Percent %

Pound (sterling) £

Prime ′

Quantifiers

Existential Quantifier

There exists, for some ∃ or ∃

There exists uniquely for exactly one ∃| or ∃|

Universal Quantifier (for all, for each, for every) ⅄ or ∀

Since (because) ∵

Tally |

Therefore

Normal ∴

Negated (it does not follow that) ∴/

Vertical Bar (end of proof) |

§159. Angstrom Unit: The angstrom unit must be treated as an abbreviation and spaced accordingly.

(1) 1/10,000 $\mu = 1$ Å 

§160. At: Except for punctuation, indicators, or symbols of grouping which apply to it, a space must be left on both sides of the *at* symbol.

(1) 3 boxes @ 27¢ 

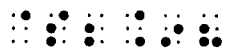
§161. Caret: A caret must be unspaced from the symbols to which it applies.

(1) .35 ∧ 73 

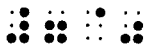
§162. Cent, Dollar, Percent, Pound (Sterling): In transcribing these signs, the corresponding symbols must occupy the same position, left or right, relative to the symbols to which they apply as is the case in ink print. These symbols must be unspaced from the symbols to which they apply.

(1) 10¢ 

(2) x¢ 

(3) $2.98 

(4) $x 

(5) 7% 

(6) x% 

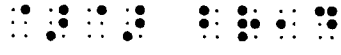
(7) £5 

(8) £x 

§163. Check Mark: Multiple check marks must be written unspaced from each other. Except for punctuation, indicators, or symbols of grouping which apply to it, a space must be left before and after a single check mark or a sequence of two or more check marks.

(1) ✓ milk 

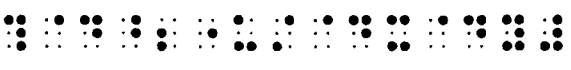
(2) (✓) eggs 

(3) ✓ ✓ bread 

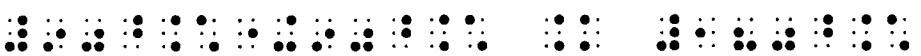
§164. Crossed d, Crossed h, Crossed Lambda, Crossed R, Partial Derivative: These symbols must be unspaced from each other and from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

(1) $R_f(P_1P_2,\ P_3P_4)$ 

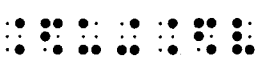
(2) $\frac{\partial}{\partial x}f(x, y)$ 

(3) $\frac{\partial^2 u}{\partial x\ \partial y}$ 

§165. Degrees: When the hollow dot is used with the meaning *degrees,* its position at the superscript level must be indicated in the transcription.

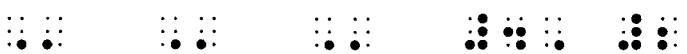
(1) $90° + 90° = 180°$ 

§166. Del: When del is used as an omission symbol, the spacing required is the same as the symbol it replaces. Otherwise, the del is subject to the spacing rule of §164.

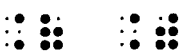
(1) $\nabla u + \nabla v$ 

§167. Ditto Mark: Ditto marks must be centered below the material which they duplicate. Except for punctuation, indicators, or symbols of grouping which apply to it, a space must be left on both sides of a ditto mark.

(1) 2 goes into 2, 1 

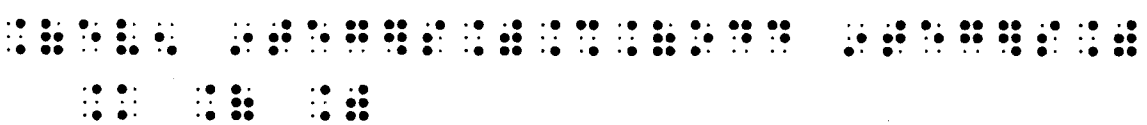
" " " 4, 2 

" " " 6, 3 

§168. Empty Set (null set, void set): The transcriber must not mistake the zero with a vertical or oblique bar (ϕ or $\not\emptyset$), meaning *empty set,* for the lower-case Greek letter phi (ϕ) to which it is similar. When the empty set is represented by the zero with a vertical or oblique bar in ink print, the corresponding braille symbol  must be used and is subject to the spacing rule of §164. When facing braces are used to represent the empty set in ink print, the corresponding braille symbols  must be used and these are spaced as grouping symbols.

(1) $A \cup \not\emptyset = A$ 

(2) {even integers} $\cap$ {odd integers} $= \{\}$



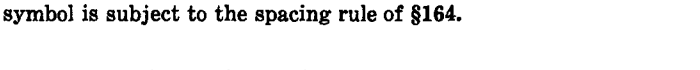
§169. **Factorial:** The factorial symbol must be unspaced from the quantity to which it applies.

(1) $n!$

(2) $(n - k)!$

§170. **Infinity:** The infinity symbol is subject to the spacing rule of §164.

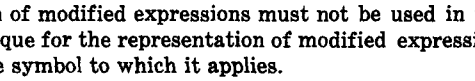
(1) $\lim_{x \to \infty} f(x)$



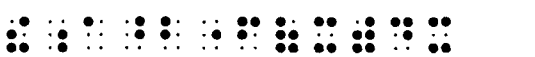
(2) $\int_0^{\infty} f(x)\,dx$

§171. **Integral:** The bar over the integral sign, or the bar under the integral sign, must be transcribed as shown in the above list. The technique for the representation of modified expressions must not be used in these cases; other modifiers, however, must be transcribed in accordance with the technique for the representation of modified expressions (see **Rule XIV**). The integral, modified or unmodified, must be unspaced from the symbol to which it applies.

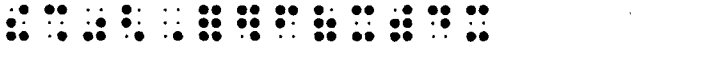
(1) $\int_a^b f(x)\,dx$



(2) $\int_0^{\infty} f(x)\,dx$



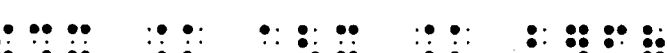
(3) $\overline{\int}_a^b f(x)\,dx = 0$



(4) $\underline{\int}_a^b f(x)\,dx = 0$

(5) $\int_{x=a}^{x=b} f(t)\,dt$

(6) $\iint_R f(x, y)\,dA$

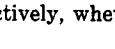


(7) $\iiint_R f(x, y, z)\,dV$

§172. **Prime:** The single and double primes are often used to denote *feet* and *inches*, respectively. They are also used to denote *minutes* and *seconds*, respectively, whether of time or of angle. Prime symbols must be unspaced from each other, and from the quantity to which they apply.

(1) x'

(2) x''



(3) x'''

(4) x'^2

(5) x'_1

(6) x'_1

(7) $(u + v)' = u' + v'$

(8) $\overline{x}'$

(9) 5′8″

(10) 20°30′10″

§173. Quantifiers: The existential and universal quantifiers must be unspaced from the quantities to which they apply.

(1) $\exists x,\ x < \frac{1}{n}$

(2) $\exists|_x \mid x = -x$

(3) $\forall x \in A$

(4) $\forall_x \forall_y -\frac{y-x}{x+y} = \frac{x-y}{x+y}$

§174. Since, Therefore: Except for punctuation, indicators, and grouping symbols, the symbol for *since* and symbols for *therefore*, in its normal or negated form, must be spaced from the material to which they apply.

(1) $\because AB = AC$

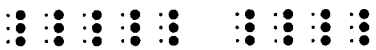
(2) $\therefore AB = AC$

§175. Tally Marks: Tally marks must be grouped in braille as they are grouped in ink print. However, the cross tally which sometimes appears in ink print must be treated as just another tally mark. Groups of tally marks must be separated by a single space from each other and, except for punctuation, indicators, and grouping symbols, from surrounding material. However, transition to another braille line takes the place of this required space. Transition to another line of braille must never be made from one tally mark to another within the same group.

(1) |||

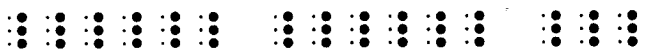
(in ink print, there is no cross tally)

(2) 𝍸 ||||



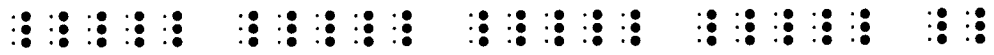
(in ink print, the first group of tallies has a cross tally)

(3) |||||| |||||| |||



(in ink print, there are no cross tallies)

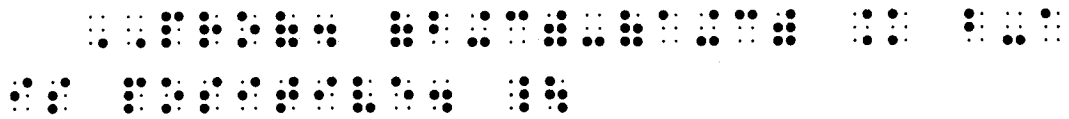
(4) 𝍸 𝍸 𝍸 𝍸 ||



(in ink print, the first four groups of tallies have cross tallies)

§176. **Boldface Vertical Bar:** The single boldface vertical bar meaning *end of proof* must be spaced from any surrounding material.

(1) PROOF. (b + c) — (a + c) = b — a is positive. **|**



RULE XXIII—MULTIPURPOSE INDICATOR

Multipurpose Indicator

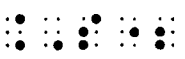
§177. **Use of the Multipurpose Indicator:** The multipurpose indicator must be used in the situations below for the specific purposes described and, when used in these situations, it must not be regarded as the base-line indicator:

i. The multipurpose indicator must be used before a modified expression as an indication to the reader of impending modification. See **Rule XIV** for additional information and examples.

ii. The multipurpose indicator must be used between a letter and a succeeding numeric symbol to indicate that the corresponding numeral is not a subscript to the corresponding letter. However, when the letter represents a numeral in a numeration system to a base other than 10, it must be regarded as a numeral and, accordingly, the multipurpose indicator must not be used.

(1) x5

(2) x.6

(3) $\Sigma 2$ 

(4) T1E4 

(a base-12 numeral; in ink print, T and E are capitalized)

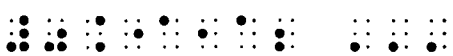
iii. The multipurpose indicator must be used between a numeric subscript and a numeral, if the latter is on the base line.

(1) $c_0 10^2 + c_1 10 + c_2$ 

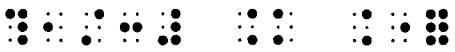
(2) $2n_1 5^{-3/2} - n_2 5^{-1/2}$

iv. The multipurpose indicator must be used between two symbols of comparison to indicate that the corresponding signs of comparison are printed horizontally and not vertically (see §149).

v. The multipurpose indicator must be used after the decimal point symbol to indicate that the symbol which follows it is not numeric unless that symbol is the comma or the punctuation indicator.

(1) $0.a_1 a_2 \ldots$ 

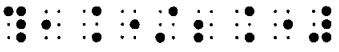
(2) $0.\alpha_1 \alpha_2 \ldots$ 

(3) $\frac{1}{3} = .\underline{?}$ 

(the general omission symbol represents a question mark over a dash in ink print)

(4) $3. + .4 = 3.4$ 

(5) (3.) 

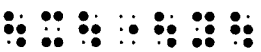
(6) $\frac{1.}{2.}$ 

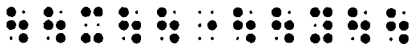
vi. The multipurpose indicator must be used between a tally mark and the punctuation indicator.

(1) 𝍸, |||| · 

(in ink print, the first group of tallies has a cross tally)

vii. The multipurpose indicator must be used between two vertical bars of which the first is a closing grouping symbol and the second is an opening grouping symbol. It must also be used between two vertical bars which are grouping symbols of which one is shorter and/or thicker than the other.

(1) $|x||y|$ 

(2) $\|x\|\|y\|$ 

(3) $\|x\|$ 

(4) $|x|_{x=0}$ 

viii. The multipurpose indicator must be used between an operation symbol when it is represented by a symbol for a regular polygon and a numeral which follows.

(1) $9 \square 14 = 23$

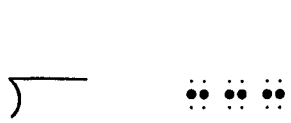
(2) $9_7 \blacksquare 13_7$ 

ix. The multipurpose indicator must be used between two symbols for the tilde to indicate that they are written horizontally, one after the other.

(1) $\sim \sim T$

RULE XXIV—SPATIAL ARRANGEMENTS

Division

Curved Division Sign on Left, Separation Line Above 

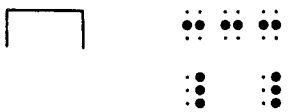
Curved Division Sign on Right, Separation Line Above 

Curved Division Signs on Left and Right, Separation Line Above 

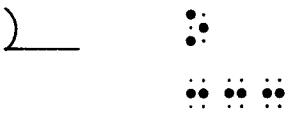
Straight or Slant Division Sign on Left, Separation Line Above 

Straight or Slant Division Sign on Right, Separation Line Above 

Straight Division Signs on Left and Right, Separation Line Above

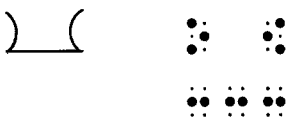


Curved Division Sign on Left, Separation Line Below

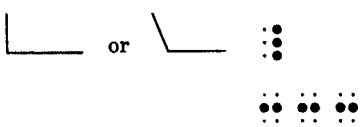


Curved Division Sign on Right, Separation Line Below

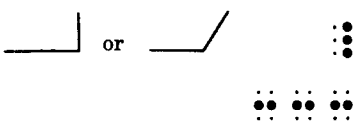
Curved Division Signs on Left and Right, Separation Line Below



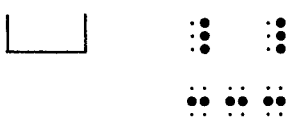
Straight or Slant Division Sign on Left, Separation Line Below



Straight or Slant Division Sign on Right, Separation Line Below



Straight Division Signs on Left and Right, Separation Line Below



Vertical Line Used in Division Arrangements (varying in length)

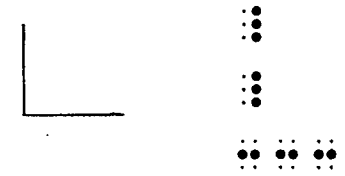
Separation Line (varying in length)

Carried Number Indicator for Addition (varying in length)

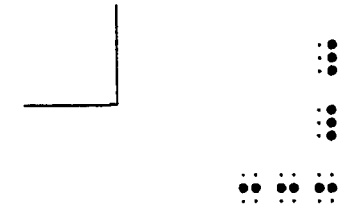
Square Root

Synthetic Division

Straight Line on Left, Separation Line Below



Straight Line on Right, Separation Line Below



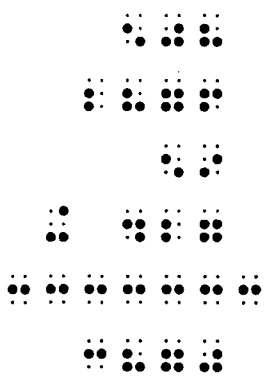
§178. Addition and Subtraction:

a. In a spatial arrangement for addition or subtraction, the numeric symbols, fractions, abbreviations, interior signs of operation or comparison must be vertically aligned with digits under digits, commas under commas, decimal points under decimal points, fractions under fractions, abbreviations under abbreviations, signs of operation under signs of operation, and signs of comparison under signs of comparison. However, if these are deliberately misaligned in ink print as in an exercise requiring the student to make a suitable correction, this misalignment must be preserved in the transcription.

b. The plus, minus, or dollar symbols, if the corresponding signs are present, must be placed at least one column of cells to the left of the widest column of numeric symbols which appears in the part of the arrangement *above* the separation line. Subject to the rules above, symbols of operation and dollar symbols may be placed in the same position as shown in ink print.

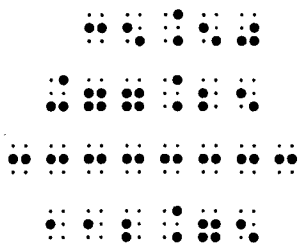
c. The separation line which appears in addition or subtraction must be made one cell longer at either end than the over-all width of the rest of the arrangement.

(1)
```
  508
 2876
   59
+ 427
------
 3870
```



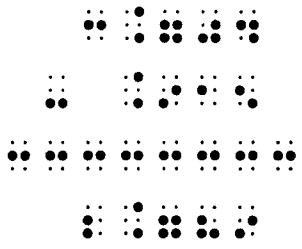
(in ink print, the plus sign is further to the left than any term in the problem or the answer)

(2)
$$\begin{array}{r} 35.50 \\ +\ 77.25 \\ \hline 112.75 \end{array}$$



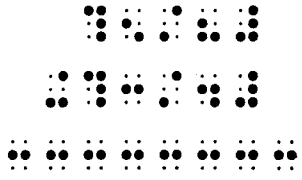
(in ink print, the plus sign is further to the left than any term in the problem or the answer)

(3)
$$\begin{array}{r} 3.704 \\ -.915 \\ \hline 2.789 \end{array}$$



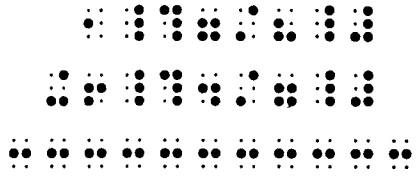
(in ink print, part of the minus sign falls under the 3, the rest extends further to the left)

(4)
$$\begin{array}{r} \frac{5}{8} \\ +\ \frac{3}{4} \\ \hline \end{array}$$



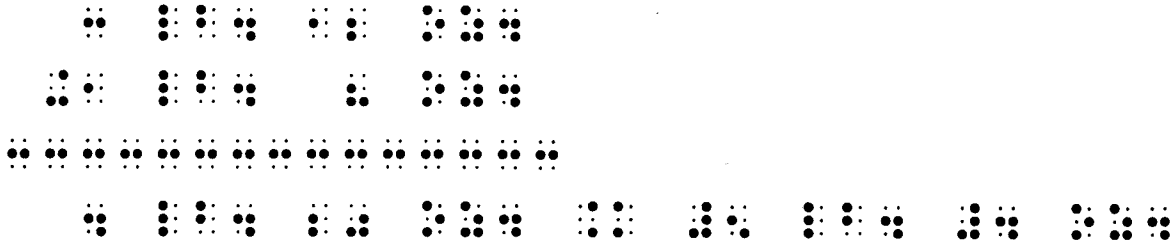
(in ink print, the plus sign is further to the left than any term in the problem)

(5)
$$\begin{array}{r} 1\ \frac{7}{8} \\ +6\ \frac{4}{7} \\ \hline \end{array}$$



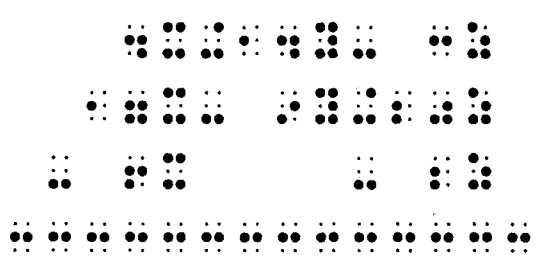
(in ink print, the plus sign is further to the left than any term in the problem)

(6)
$$\begin{array}{r} 3 \text{ lb. } 12 \text{ oz.} \\ +\ 1 \text{ lb. } 8 \text{ oz.} \\ \hline 4 \text{ lb. } 20 \text{ oz.} = 5 \text{ lb. } 4 \text{ oz.} \end{array}$$



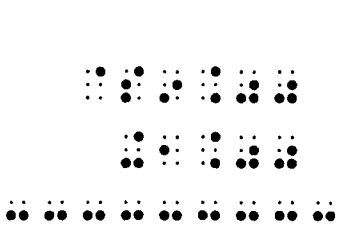
(in ink print, the plus sign is further to the left than any term in the problem)

(7)
$$\begin{array}{rrrrrr} 4x & + & 14y & - & 3z \\ 17x & - & 9y & + & 20z \\ -\ 6x & & & - & 2z \\ \hline 15x & + & 5y & + & 15z \end{array}$$



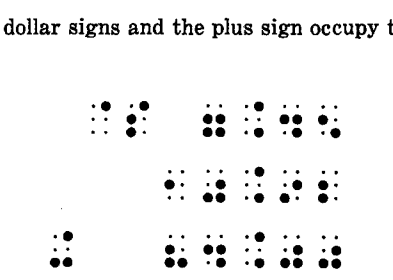
(in ink print, the minus sign is further to the left than any term in the problem)

(8)
$$\begin{array}{r} \$9.00 \\ +1.00 \\ \hline \$10.00 \end{array}$$



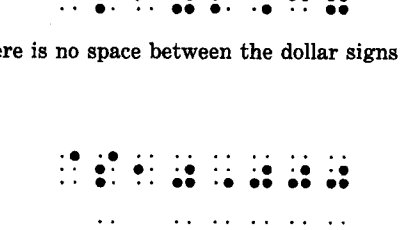
(in ink print, the dollar signs and the plus sign occupy the same position as in braille)

(9)
$$\begin{array}{r} \$7.45 \\ 10.92 \\ +84.00 \\ \hline \$102.37 \end{array}$$



(in ink print, there is no space between the dollar signs and the following digits, and the plus sign is to the left of the first dollar sign)

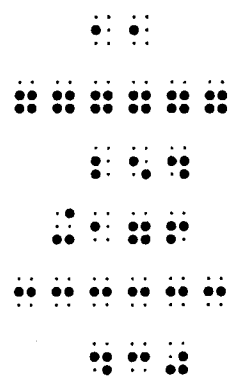
(10)
$$\begin{array}{r} \$10,000 \\ -\ 9,000 \\ \hline \$\ 1,000 \end{array}$$



(in ink print, the dollar signs and the minus sign occupy the same position as in braille)

d. Carried Numbers in Addition: When carried numbers appear in an addition arrangement above the columns to which they apply, the transcriber must insert the indicator for carried numbers between these carried numbers and the arrangement to which they apply. The carried number indicator must have the same length as the separation line.

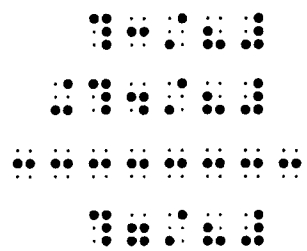
(1)
$$\begin{array}{r} {\scriptstyle 11}\\ 254\\ +176\\ \hline 430 \end{array}$$



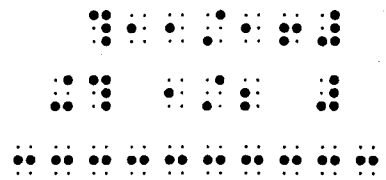
(in ink print, the carried numbers are in small type directly above the columns to which they apply)

e. In an arrangement containing fractions, fraction lines must be vertically aligned, each numerator must be right justified in the column reserved for numerators, and each denominator must be left justified in the column reserved for denominators. Fraction indicators must also be vertically aligned and must be right-justified in the columns reserved for both opening and closing indicators.

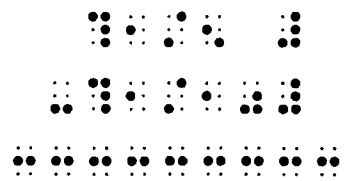
(1)
$$\begin{array}{r} \frac{3}{8}\\ +\ \frac{4}{8}\\ \hline \frac{7}{8} \end{array}$$



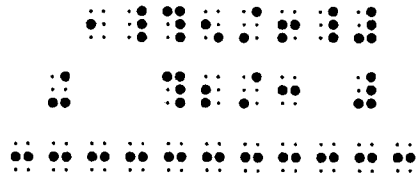
(2)
$$\begin{array}{r} \frac{11}{16}\\ +\ \frac{1}{2}\\ \hline \end{array}$$



(3)
$$\begin{array}{r} \frac{1}{5}\\ -\ \frac{1}{10}\\ \hline \end{array}$$

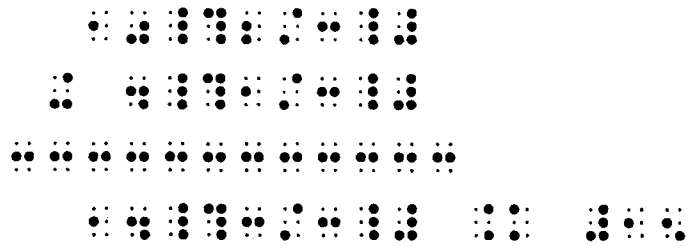


(4) $1\frac{5}{6}$
$+\frac{2}{3}$



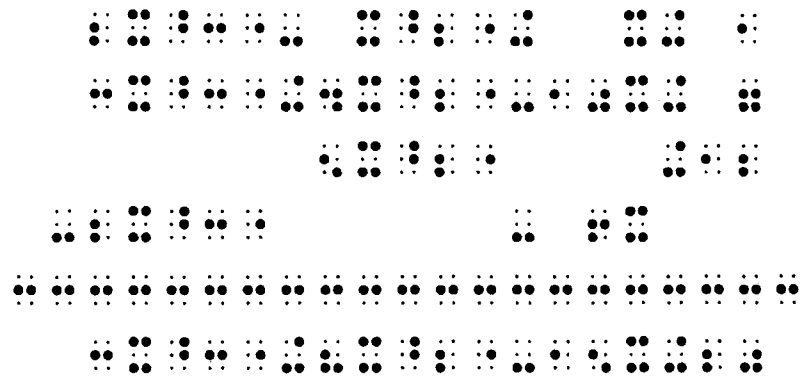
f. In an arrangement containing mixed numbers, the whole-number part must be vertically aligned according to a above.

(1) $10\frac{2}{3}$
$+\ 4\frac{1}{3}$
$14\frac{3}{3} = 15$



g. In an arrangement containing polynomials, terms must be vertically aligned. In each term, symbols of operation, coefficients, letters, superscript indicators, superscripts, and base-line indicators must also be vertically aligned. When the base-line indicator is required, it must be placed in the first possible position consistent with this required alignment. Within each coefficient and superscript, corresponding symbols must be vertically aligned.

(1)
$$\begin{array}{rrrrrrr} 2x^3 & - & x^2 & + & x & + & 1 \\ 3x^3 & + & 4x^2 & - & 10x & + & 7 \\ & & 5x^2 & & & + & 12 \\ -\ 2x^3 & & & - & 6x & & \\ \hline 3x^3 & + & 8x^2 & - & 15x & + & 20 \end{array}$$



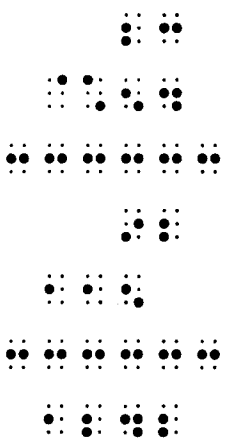
§179. **Multiplication:**

a. In a spatial arrangement for multiplication, the symbols comprising the multiplier and multiplicand must be aligned in the transcription as the corresponding signs are aligned in ink print.

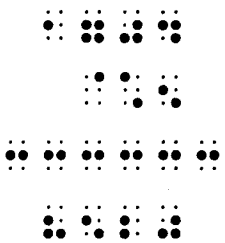
b. The multiplication symbol, if the corresponding sign is present in ink print, must be placed immediately to the left of the multiplier.

c. The separation lines which appear in a multiplication arrangement must be made one cell longer at either end than the overall width of the rest of the arrangement.

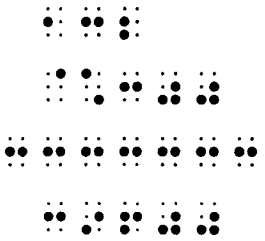
(1)
$$\begin{array}{r} 23 \\ \times 54 \\ \hline 92 \\ 115 \\ \hline 1242 \end{array}$$



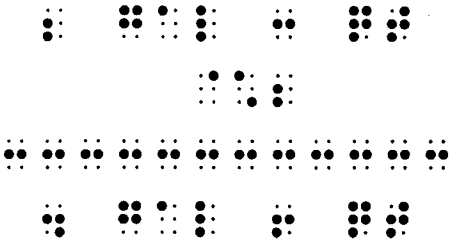
(2)
$$\begin{array}{r} 1704 \\ \times\ 5 \\ \hline 8520 \end{array}$$



(3)
$$\begin{array}{r} 132 \\ \times 300 \\ \hline 39600 \end{array}$$

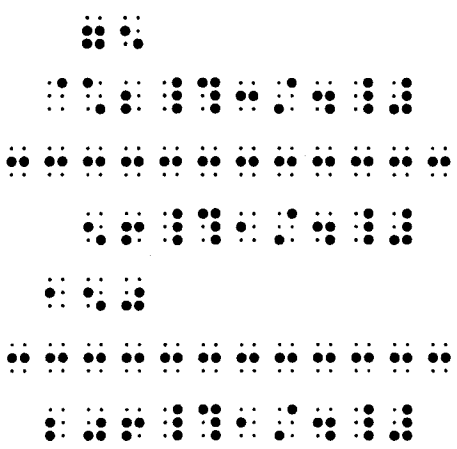


(4)
$$\begin{array}{r} 2 \text{ gal } 3 \text{ qt} \\ \times 2 \\ \hline 4 \text{ gal } 6 \text{ qt} \end{array}$$

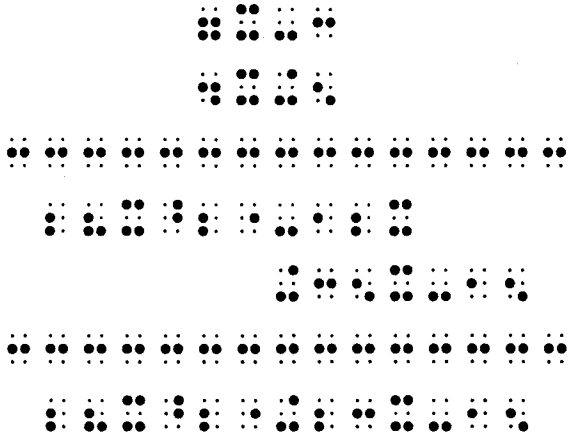


d. In an arrangement containing fractions, mixed numbers, or polynomials, alignment must be generally as specified in §178 e-g.

(1)
$$\begin{array}{r} 75 \\ \times\ 2\tfrac{1}{4} \\ \hline 56\tfrac{1}{4} \\ 150 \\ \hline 206\tfrac{1}{4} \end{array}$$

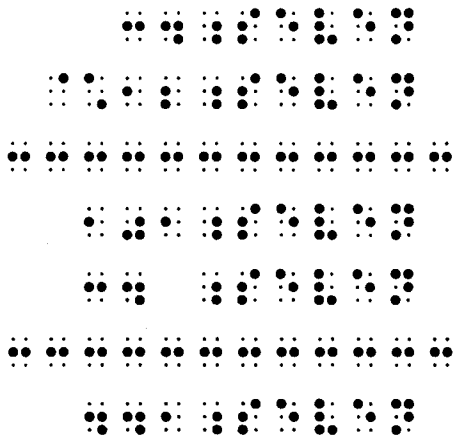


(2)
$$\begin{array}{r} 7x\ -\ 3 \\ 4x\ +\ 5 \\ \hline 28x^2\ -\ 12x \\ +\ 35x\ -\ 15 \\ \hline 28x^2\ +\ 23x\ -\ 15 \end{array}$$



e. In arrangements which show multiplication to non-decimal bases in which subscripts appear, the subscript indicator must be placed in the first possible position consistent with the alignment required for the addition of partial products.

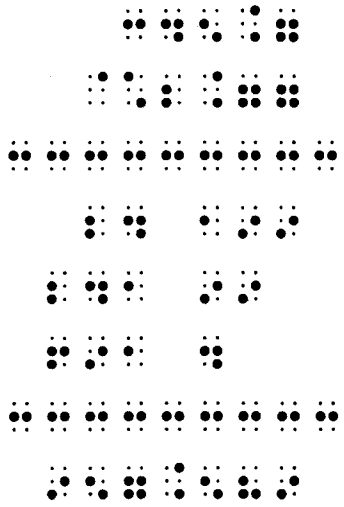
(1)
$$\begin{array}{r} 34_{\text{seven}} \\ \times\ 12_{\text{seven}} \\ \hline 101_{\text{seven}} \\ 34_{\text{seven}} \\ \hline 441_{\text{seven}} \end{array}$$



f. When commas or the decimal point occur in the answer of a multiplication arrangement, a blank column of cells must be left above these in the partial products.

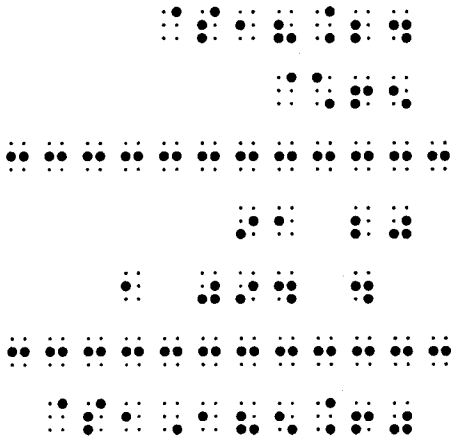
(1)

```
  345.7
 × 2.77
 -------
  24199
 24199
 6914
 -------
 957.589
```



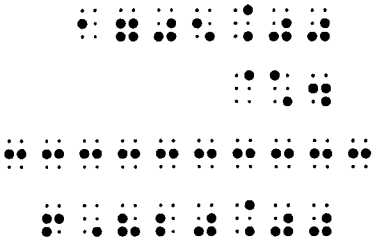
(2)

```
  $18.24
 ×  65
 -------
    9120
   10944
 -------
 $1,185.60
```



(3)

```
  1705.00
 ×    4
 -------
 6,820.00
```



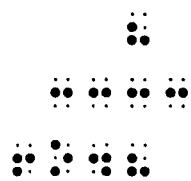
§180. Division:

a. In a spatial arrangement for division, the symbols comprising the dividend and the partial products and differences must be aligned in the same way as the corresponding signs are aligned in ink print. Symbols in the quotient must be aligned with their corresponding symbols in the dividend unless they are specifically unaligned in ink print.

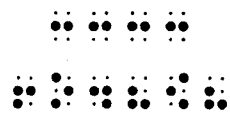
b. The division symbol must be placed in the cell directly before the dividend or directly after the dividend according as the corresponding division sign occurs in the forward or reverse direction in ink print. The divisor must be placed so that there is no space between it and the division symbol to which it applies. If a quotient also applies to a division symbol by being placed on the same line as the dividend, no space must be left between the quotient and the division symbol to which it applies. If a horizontal line occurs under a divisor as part of a division sign in ink print, this line must be ignored in the transcription.

c. Each separation line which appears in a division arrangement must begin in the column containing a division symbol and must end in the column containing the other division symbol, if the latter appears in ink print. Otherwise, each separation line must end in a cell one column beyond the overall arrangement. However, when the division arrangement contains only a divisor and a dividend, but no quotient and no partial products and differences, the separation line, whether shown above or below the dividend, must be omitted. In this case, the division arrangement must not be regarded as spatial. In particular, a blank line must not be left above or below such a division arrangement, and the numeric indicator must be used in the appropriate place.

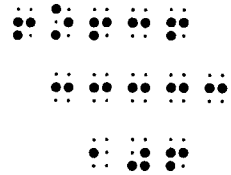
(1)
$$\begin{array}{r} 8 \\ \hline 6)48 \end{array}$$



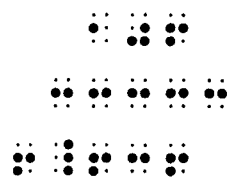
(2) $6)\overline{48}(8$



(3)
$$\begin{array}{r} 6)636 \\ \hline 106 \end{array}$$

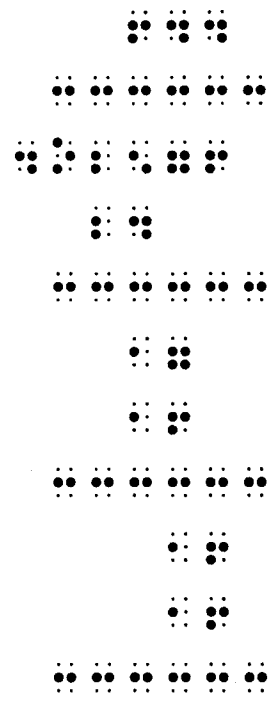


(4)
$$\begin{array}{r} 106 \\ \hline 6\,|636 \end{array}$$



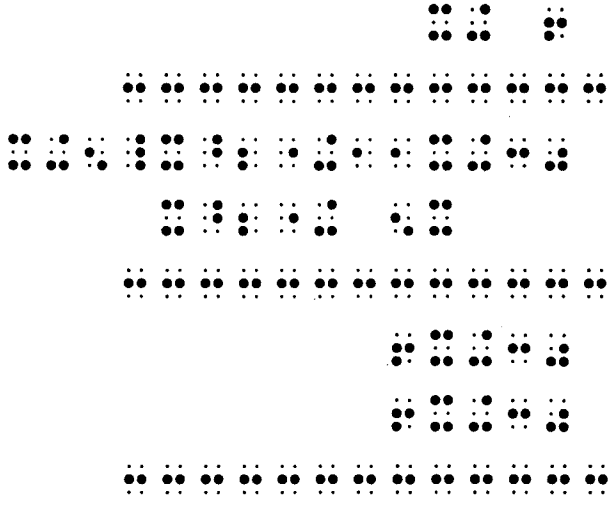
(5)

```
  644
4)2576
  24
  17
  16
   16
   16
```



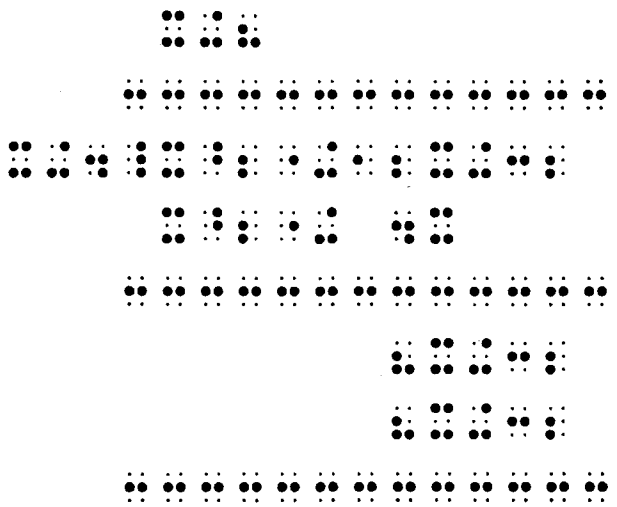
(6)

```
          x + 6
x + 5 | x² + 11x + 30
        x² + 5x
        ------
          6x + 30
          6x + 30
          -------
```



(in ink print, it is clear that the quotient is aligned with the dividend, and there is a horizontal line under the divisor)

(7)
$$\begin{array}{r} x + 8 \\ x + 4 \overline{)\, x^2 + 12x + 32} \\ x^2 + 4x \\ \hline 8x + 32 \\ 8x + 32 \\ \hline \end{array}$$

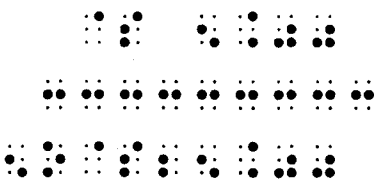


(in ink print, the quotient is clearly not aligned with the terms in the dividend)

(8) $18\overline{)452}$ 

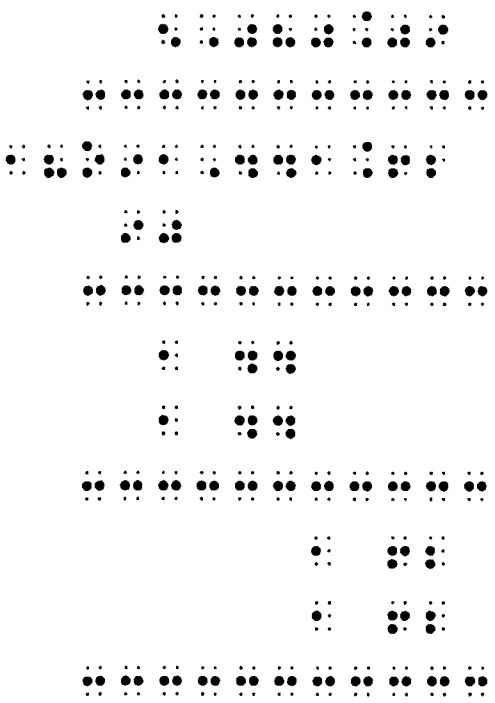
d. When commas or the decimal point occur in the dividend of a division arrangement, a blank column of cells must be left where these occur in the entire arrangement except in separation lines. When a caret occurs in a dividend, a blank column of two cells must be left where this occurs in the entire arrangement except in the separation lines and the quotient. In the quotient, the decimal point corresponding to the caret must be right-justified in the two cells allotted to the caret.

(1)
$$\begin{array}{r} \$\ 5.00 \\ 5\overline{)\$25.00} \end{array}$$



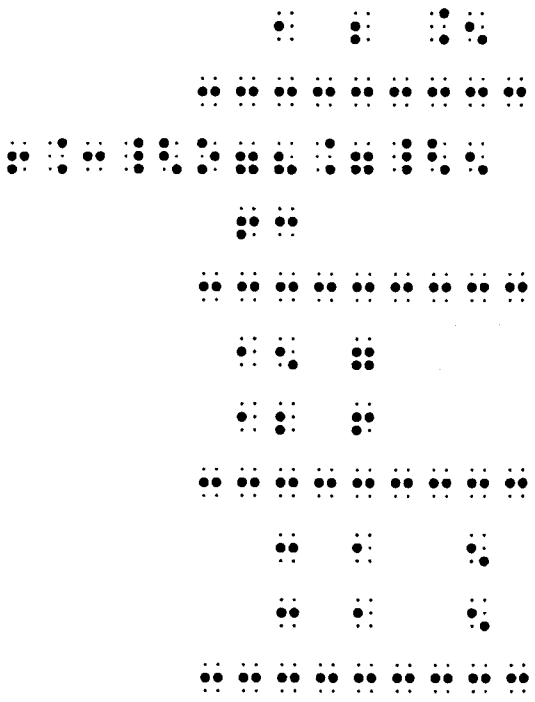
(2)

```
      5,080.09
18)91,441.62
   90
   ----
    1 44
    1 44
    ----
      1 62
      1 62
      ----
```



(3)

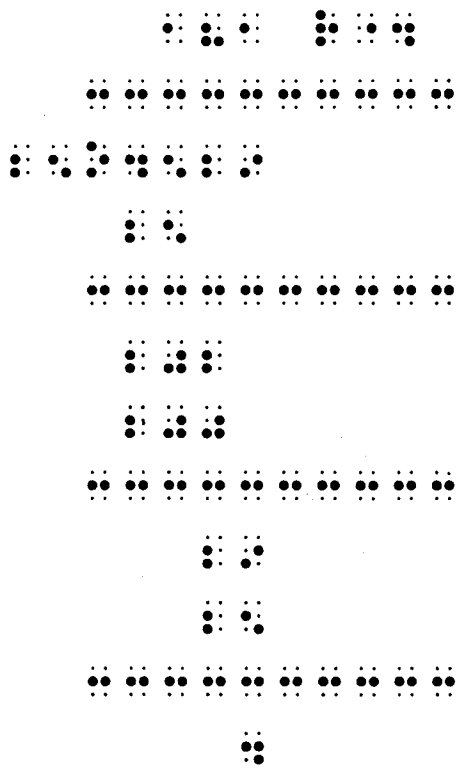
```
      1 2.5
6.3^)78.7^5
   63
   ----
    15 7
    12 6
    ----
     3 1 5
     3 1 5
     -----
```



e. When, in a division arrangement, there is a remainder which is identified as such by the letter "r", lower-case or capitalized, the "r" must be preceded by a space.

(1)

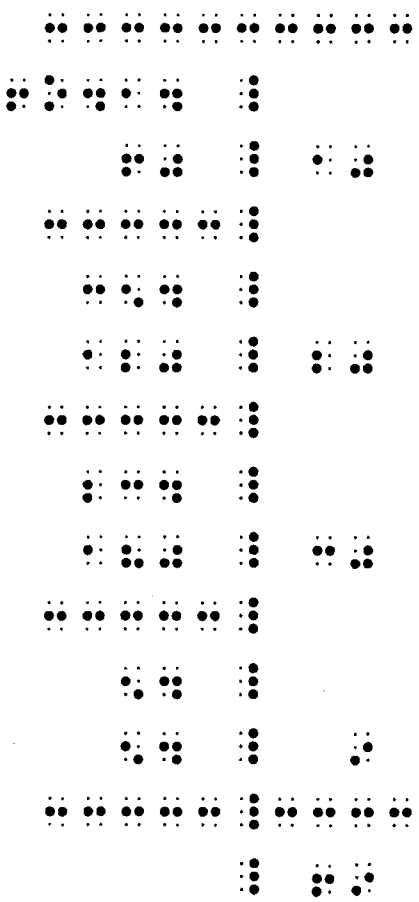
```
   181 r4
25)4529
   25
   ---
   202
   200
   ---
    29
    25
    --
     4
```



f. If a vertical line is part of a division arrangement, it may be represented by a column of dots 4-5-6 or it may be drawn. A space must be left between the column of dots 4-5-6 and any digit which precedes or follows it.

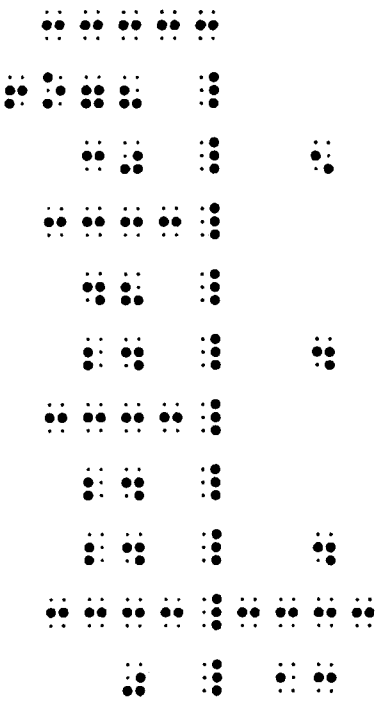
(1)

6)414	
60	10
354	
120	20
234	
180	30
54	
54	9
	69



(2)

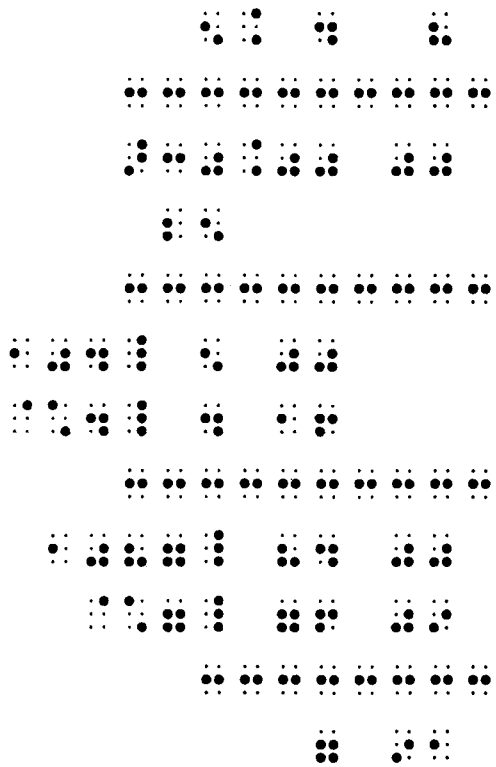
6)78	
30	5
48	
24	4
24	
24	4
0	13



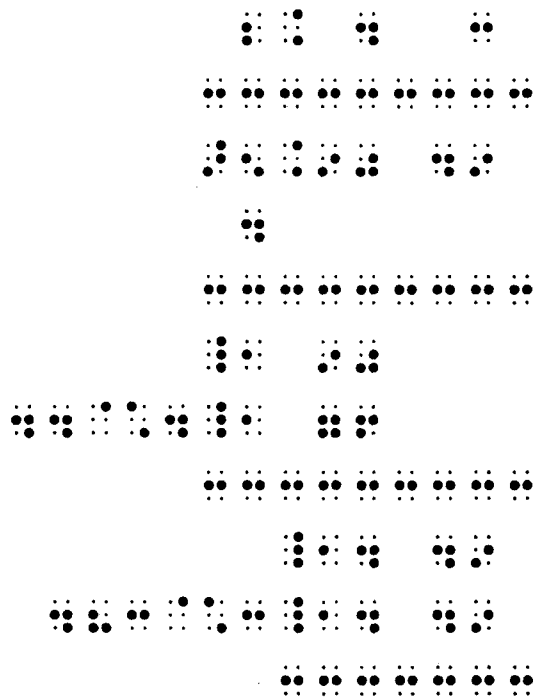
§181. Square Root: A square root arrangement is similar to a division arrangement except that no divisor is present. The arrangement should be adapted to resemble the arrangement in ink print as closely as possible. If the square root symbol is used in this situation, the termination indicator is not required.

(1)

```
      5. 4  8
√30.00 00
   25
104 | 5 00
×4  | 4 16
1087| 84 00
×7  | 76 09
      7 91
```



(2)
$$\begin{array}{r} 2.\ 4\ 3 \\ \sqrt{5.90\ 49} \\ 4 \\ \hline 1\ 90 \\ 44 \times 4 \quad 1\ 76 \\ \hline 14\ 49 \\ 483 \times 3 \quad 14\ 49 \\ \hline \end{array}$$

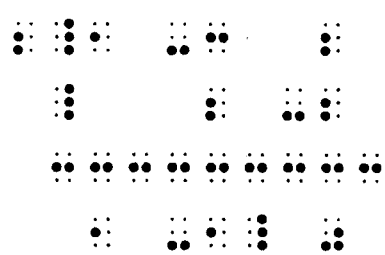


§182. Synthetic Division:

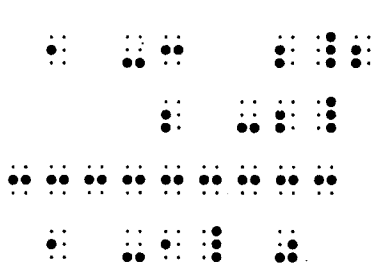
a. In a synthetic division arrangement, the numeric symbols in the synthetic dividend, synthetic product, and synthetic quotient must be aligned by place value. Symbols of operation, when present, must also be aligned. There must be at least one column of blank cells between adjacent columns of a synthetic division arrangement.

b. A vertical line must be used to the left or to the right of the synthetic division arrangement according as the synthetic divisor appears to the left or to the right. This vertical line must be unspaced from the synthetic dividend and from the synthetic divisor. One part of the vertical bar must appear on the line containing the synthetic dividend, and another part of the line must appear on the line containing the synthetic product. The separation line must begin directly under the vertical line at one end, and terminate one cell beyond the over-all synthetic arrangement at the other end. If the synthetic divisor appears in ink print as boxed-in on two sides, this must be ignored in the transcription. When a vertical line is used between the synthetic quotient and the synthetic remainder, it must be placed in the column of blank cells as shown in ink print.

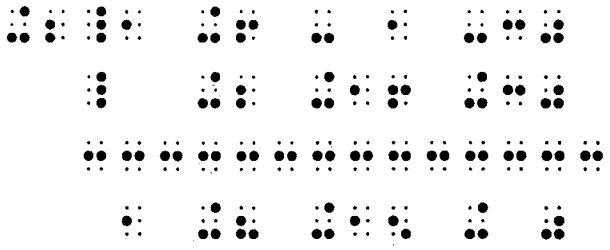
(1)
$$\begin{array}{r|rrr} 2 & 1 & -3 & 2 \\ & & 2 & -2 \\ \hline & 1 & -1 & 0 \end{array}$$



(2)
$$\begin{array}{rrrr|l} 1 & -3 & 2 & & 2 \\ & 2 & -2 & & \\ \hline 1 & -1 & 0 & & \end{array}$$

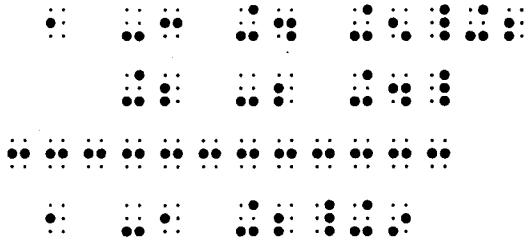


(3)
$$\begin{array}{l|rrrr} +2 & 1 & +6 & -1 & -30 \\ & & +2 & +16 & +30 \\ \hline & 1 & +8 & +15 & +0 \end{array}$$



(in ink print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

(4)
$$\begin{array}{rrrr|l} 1 & -3 & +4 & +5 & +2 \\ & +2 & -2 & +4 & \\ \hline 1 & -1 & +2 & +9 & \end{array}$$



(in ink print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

§183. Determinants and Matrices:

a. In determinants or matrices each entry must be left-justified (moved as far left as possible) in the column to which it belongs, and top-justified (moved as far up as possible) in the row to which it applies. Regardless of the ink-print copy, centering or other forms of alignment is not permitted. One column of blank cells must be left between columns.

b. Every effort must be made to confine the entire arrangement to a single braille page. To achieve this goal, the following techniques may be used:

i. An entry may be run over to other braille lines and each continuation indented two cells from the column margin. When space saving is a factor, runovers may be made without regard to any hierarchy preferences. Successive rows in a column must be transcribed without skipping a line between them.

ii. An entry may be run over to other braille lines and each continuation left-justified in its column. When space saving is a factor, runovers may be made without regard to any hierarchy preferences. Successive rows in a column must be transcribed with a

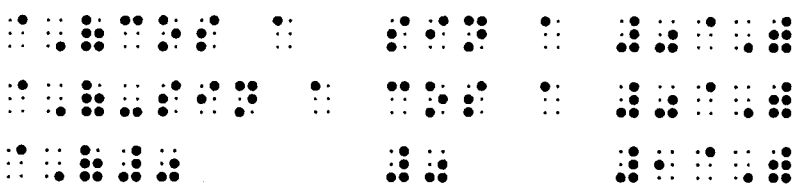
skipped line between them. When the technique described in i above is effective in providing the required space, it must be used in preference to the technique described here.

iii. Additional space may be saved by drawing the enclosing grouping symbols instead of using their braille equivalents.

iv. When an entry is a fraction, the fraction may be represented spatially, if necessary, to save space. However, the row containing such a fraction must then have a line skipped above and below it.

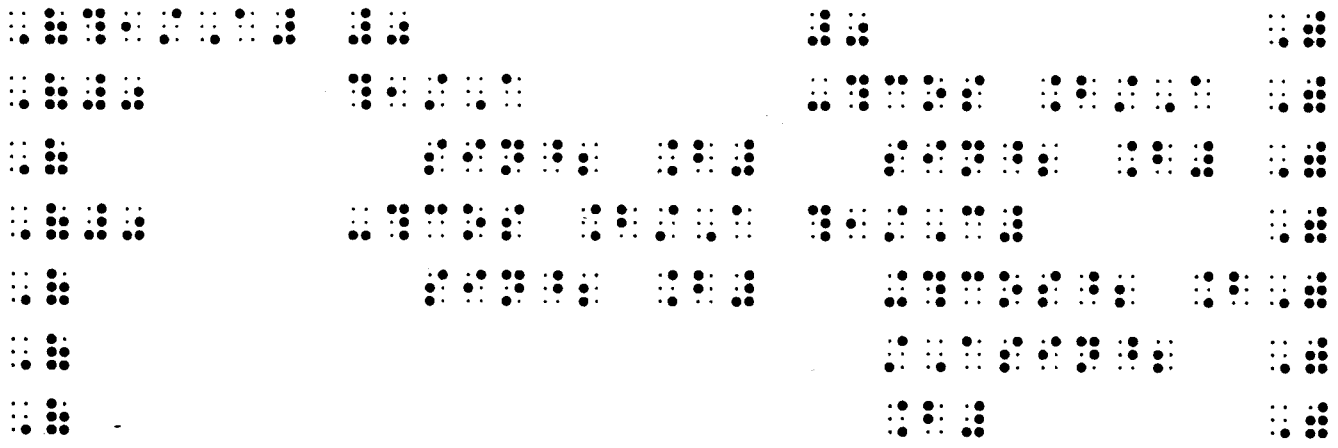
v. The technique of keying may be employed for one or several entries if no other space-saving technique is effective. (See §187.)

(1)
$$\begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



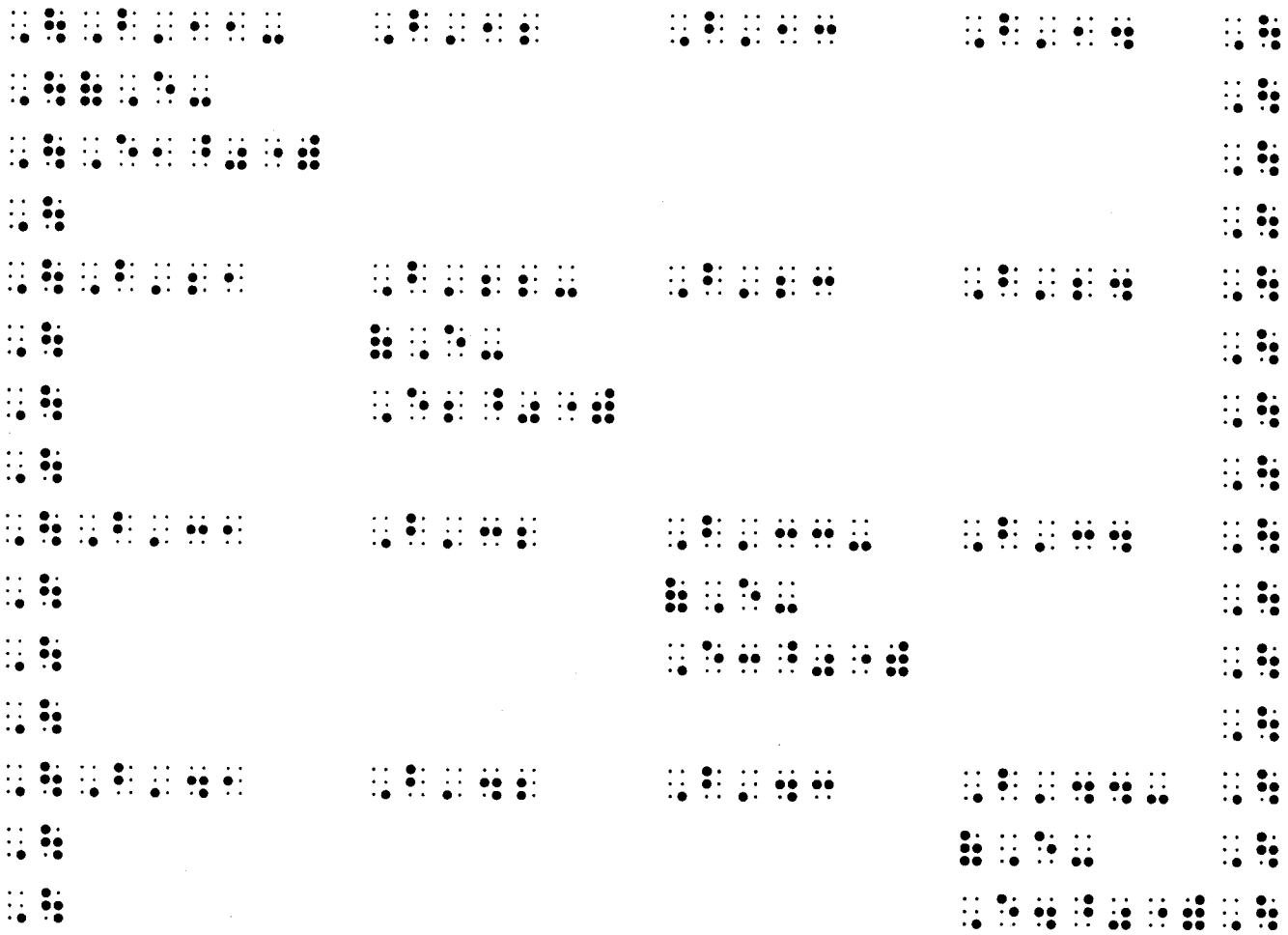
(in ink print, each entry is centered in the column to which it belongs)

(2)
$$\begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A \sin^2 \beta} & -\frac{\cos \beta}{A \sin^2 \beta} \\ 0 & -\frac{\cos \beta}{A \sin^2 \beta} & \frac{1}{C} + \frac{\cos^2 \beta}{A \sin^2 \beta} \end{pmatrix}$$



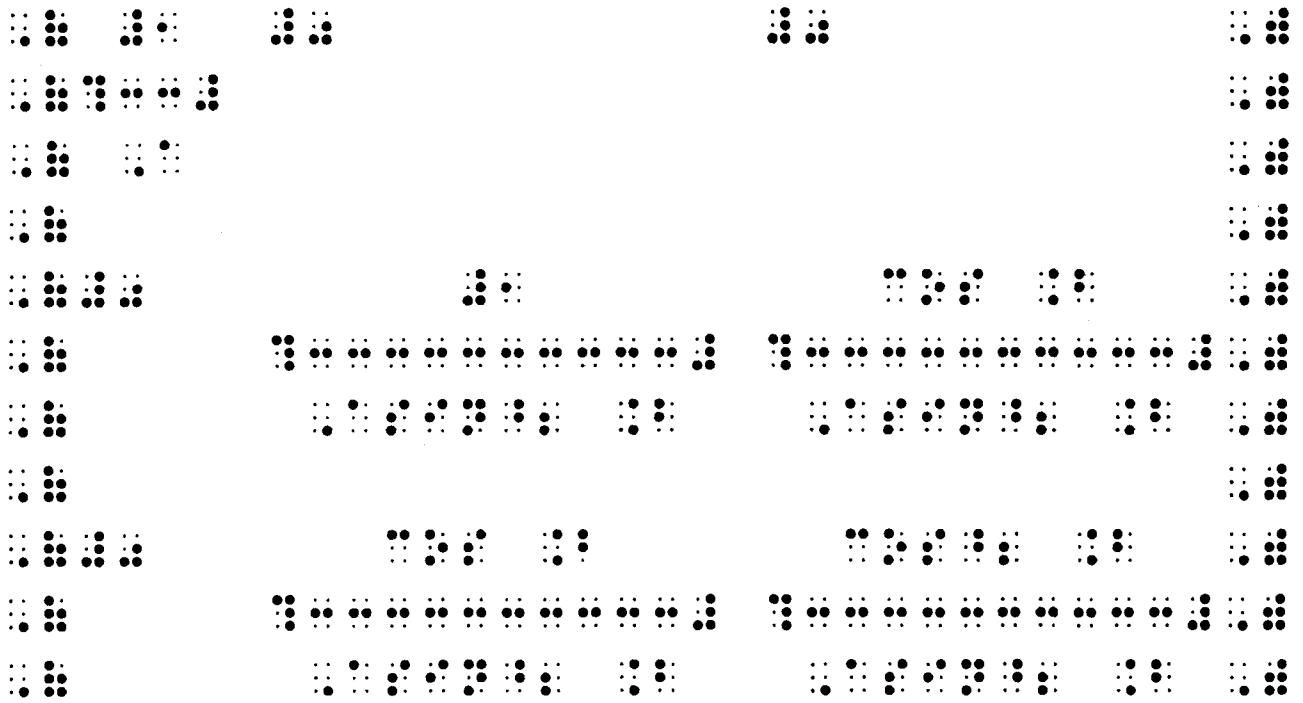
(in ink print, each entry is centered in the column to which it belongs)

$$(3)\quad \begin{vmatrix} B'_{11} - (E - E^0_1) & B'_{12} & B'_{13} & B'_{14} \\ B'_{21} & B'_{22} - (E - E^0_2) & B'_{23} & B'_{24} \\ B'_{31} & B'_{32} & B'_{33} - (E - E^0_3) & B'_{34} \\ B'_{41} & B'_{42} & B'_{43} & B'_{44} - (E - E^0_4) \end{vmatrix}$$



(in ink print, each entry is centered in the column to which it belongs)

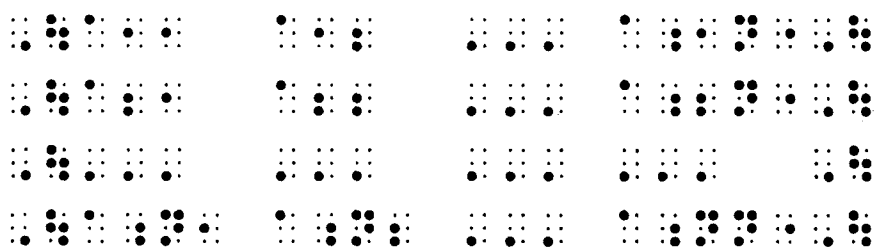
(4) $$\begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A \sin^2 \beta} & \frac{\cos \beta}{A \sin \beta} \\ 0 & \frac{\cos \beta}{A \sin^2 \beta} & \frac{\cos^2 \beta}{A \sin^2 \beta} \end{pmatrix}$$



(in ink print, each entry is centered in the column to which it belongs)

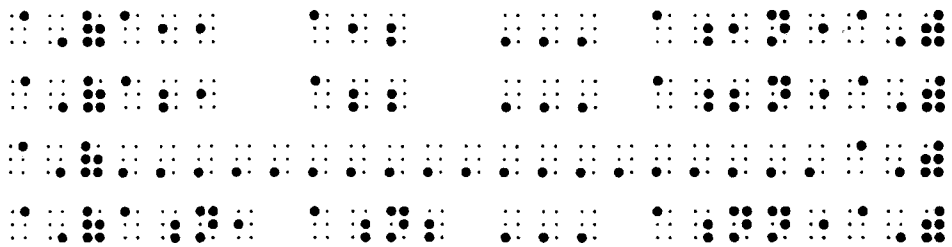
c. When a sequence of dots appears to signify the omission of one or more rows and such dots are confined to each column of the determinant or matrix, a sequence of three dots 3 must be placed in each column to indicate the omission and each ellipsis used in this way must be left-justified in its column. When a sequence of dots appears to signify the omission of one or more rows and such dots are not confined to their columns, or if some columns contain no dots, a sequence of dots 3 must be used beginning in the first cell of column one and extending to the end of the longest entry in the last column.

(1) $$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & \dots & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$



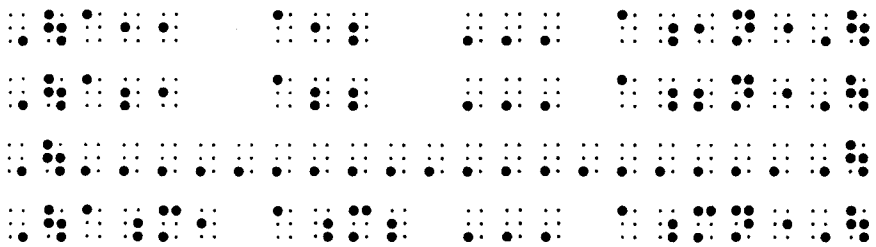
(in ink print, one dot is shown in the first, second, and fourth columns, respectively)

(2) $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$



(in ink print, the sequence of dots in the third row is not confined to specific columns)

(3) $\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}$

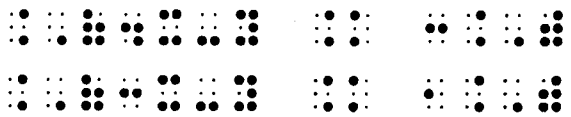


(in ink print, three dots are vertically aligned, one under the other in the first and fourth columns; no such dots appear in the second or third columns)

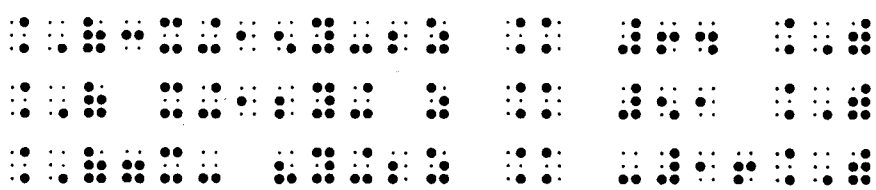
§184. Unified Expressions:

a. When enlarged grouping symbols are used to unify an expression which is neither a determinant nor a matrix, each item must begin in the cell which immediately follows the left enlarged grouping symbol and must end in the cell which immediately precedes the right enlarged grouping symbol. It is advantageous to draw these enlarged grouping symbols when space saving is a factor. However, these requirements must be waived whenever vertical alignment must be indicated. In this case, at least one item must either begin in the cell which immediately follows the left enlarged grouping symbol or must end in the cell which immediately precedes the right enlarged grouping symbol.

(1) $\left\{ \begin{array}{l} 4x - y = 3 \\ 3x - y = 1 \end{array} \right\}$

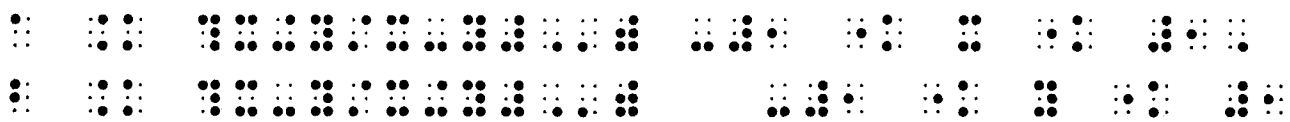


(2) $\left\{ \begin{array}{l} 3x + 15y - 2z = 64 \\ x + 12y + z = 51 \\ 7x - 8y + 2z = -16 \end{array} \right\}$



b. When an explanation or comment refers to more than one ink print line to which no grouping sign as a whole applies, the implied grouping must be indicated by using a transcriber's enlarged grouping symbol. If the explanation occurs to the left, the left transcriber's enlarged grouping symbol must be used; if it occurs to the right, the right transcriber's enlarged grouping symbol must be used. There must be at least one clear column of spaces between either of these transcriber's enlarged grouping symbols and the associated explanation. If the explanation requires more braille lines than what is being explained, the transcriber's enlarged grouping symbol must be extended to cover the explanation, and each runover of the explanation must be indented two cells from the column in which the explanation begins.

(1) $a = \frac{x + y}{x - y}$ $b = \frac{x - y}{x + y}$ $-1 < x < 1, -1 < y < 1$



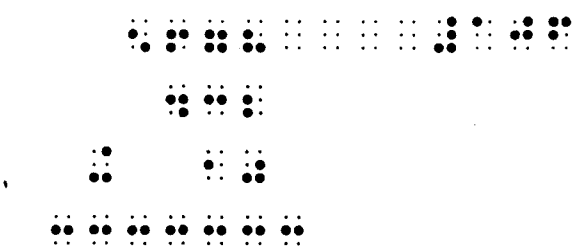
(in ink print, the explanation is centered to the right of the two equations to which it applies)

RULE XXV—FORMAT

§185. Spatial Arrangements:

a. When the transcription is in the form of a spatial arrangement, a blank line must be left both above and below the spatial arrangement even if the spatial arrangement directly precedes or follows the page-change line indicating a new ink print page. Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line. However, when a running head is used, a line must be skipped between the running head and a spatial arrangement. When a spatial arrangement begins on the first or second line of a braille page or ends on the twenty-fourth or the twenty-fifth line of a braille page, there must always be at least three clear columns of cells between the last symbol on any line of the arrangement, including any separation lines, and the first symbol of a page number. If this cannot be achieved, the arrangement must begin on line 3 or end on line 23, respectively. The entire spatial arrangement should be confined to one braille page.

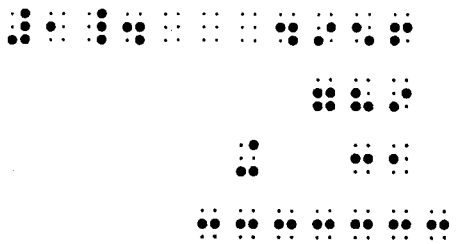
(1)
```
  5678    106
   432
 + 10
-----
```



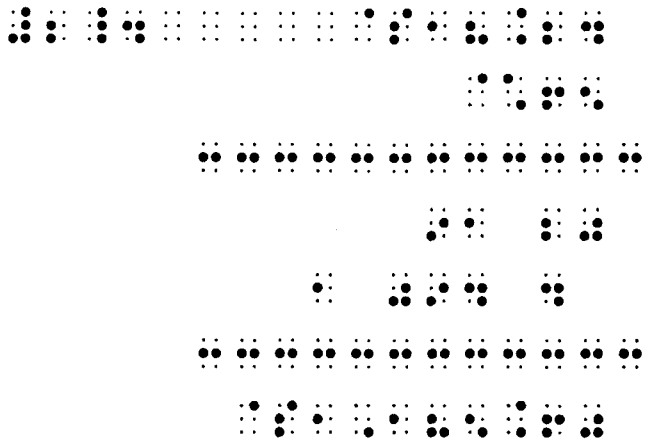
b. When a spatial arrangement is identified by a number or a letter, such as in a set of exercises, the identifier must be placed as indicated below. In all cases, there must be one column of blank cells between the identifier and the left-most symbol of the arrangement as a whole, including any separation lines.

i. The identifier must be placed on the top line of an addition, subtraction, or multiplication arrangement. However, when the carried-number indicator has been used in an addition arrangement or when numbers have been canceled out in a subtraction arrangement, the identifier must be placed on the line which contains the first term of the addition arrangement or the minuend of the subtraction arrangement.

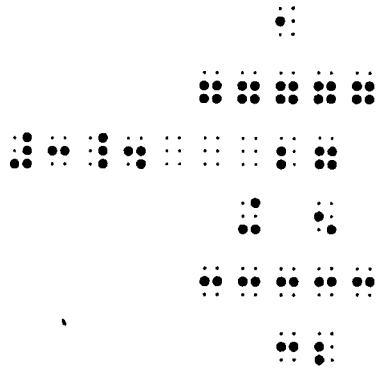
(1) 1.
$$\begin{array}{r} 4956 \\ 789 \\ +\ 31 \\ \hline \end{array}$$



(2) 2.
$$\begin{array}{r} \$18.24 \\ \times 65 \\ \hline 9120 \\ 10944 \\ \hline \$1,185.60 \end{array}$$

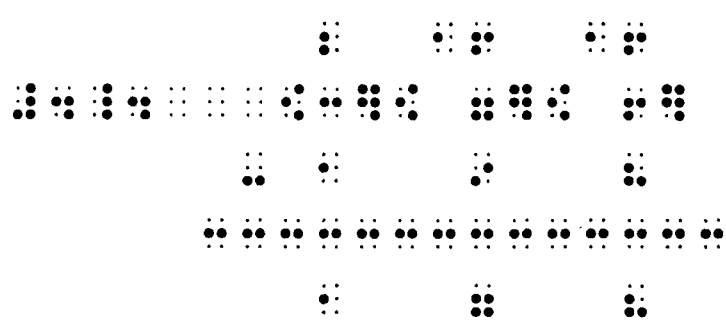


(3) 3.
$$\begin{array}{r} {\scriptstyle 1} \\ 27 \\ +\ 5 \\ \hline 32 \end{array}$$



(4)

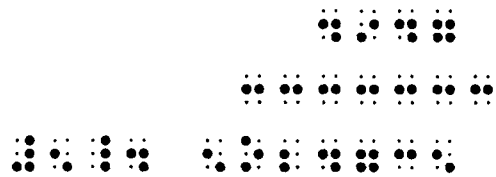
4.
$$\begin{array}{r} {\scriptstyle 2}\ {\scriptstyle 16}\ {\scriptstyle 16} \\ \not{8}\ \not{7}\ \not{6} \\ -\ 1\ 9\ 8 \\ \hline 1\ 7\ 8 \end{array}$$



ii. The identifier must be placed on the line which contains the dividend in a division arrangement, on the line which contains the radicand in a square root arrangement, and on the line which contains the synthetic dividend in a synthetic division arrangement.

(1)

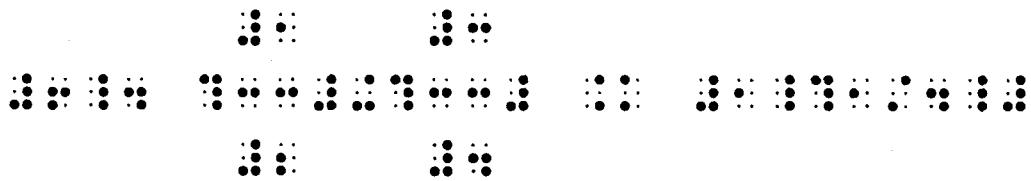
5.
$$5\overline{)24735}\quad 4947$$



iii. In the case of spatial fractions, identifiers and centered comparison symbols, symbols of operation, punctuation, and other applicable symbols must be placed on the principle fraction line. However, identifiers must be placed on the top line of a continued fraction.

(1)

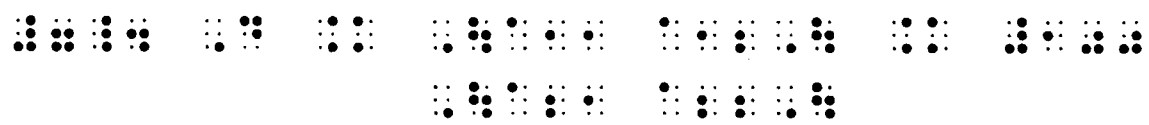
6. $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$



iv. In the case of determinants, matrices, and unified expressions, identifiers, comparison symbols, symbols of operation, punctuation, and other applicable symbols must, if they appear on the same side of the expression as the enlarged grouping symbol, be placed on the top line even though they are centered in ink print.

(1)

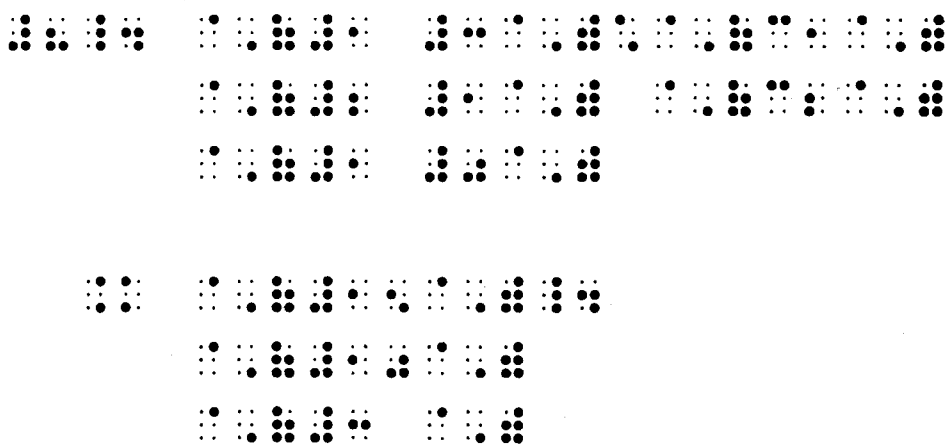
7. $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 100$



(in ink print, the material outside the determinant is centered)

(2)

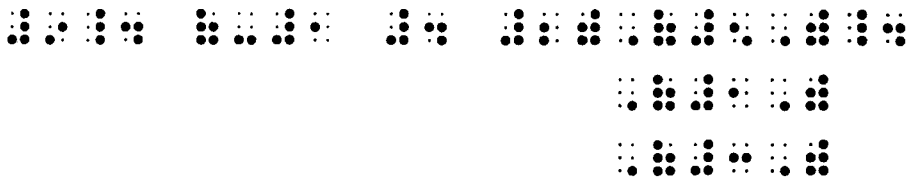
8. $\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \\ 3 \end{bmatrix}$.



(in ink print, the example number, the multiplication dot, the second matrix, the equals sign, and the period are all centered vertically in relation to the first and last matrices)

(3)

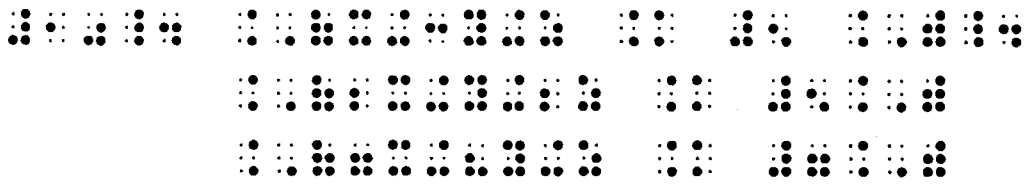
9. $(-1 \quad 4 \quad 2)\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$.



(in ink print, the example number, first matrix, and period are vertically centered in relation to the second matrix)

(4)

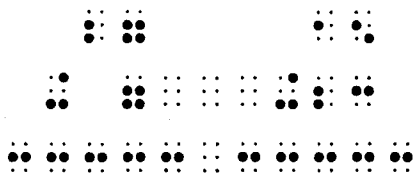
10. $\left\{\begin{array}{l} x + 3y + z = 5 \\ 2x + y + 2z = 5 \\ 7x + 8y + z = 7 \end{array}\right\}$.



(in ink print, the example number and the period are vertically centered in relation to the unified system)

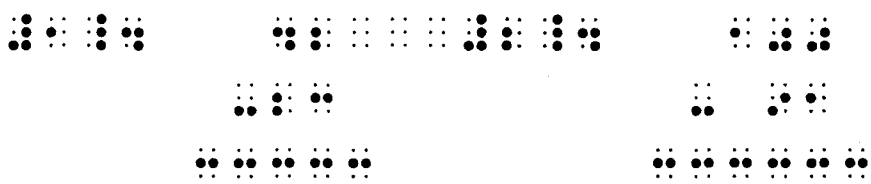
c. When spatial arrangements are placed side-by-side there must be at least one clear column of blank cells between the end of one separation line and the beginning of the next. In any case, no symbol in one spatial arrangement may be less than three cells distant from any symbol on any line in, or associated with, a neighboring arrangement other than neighboring ends of separation lines.

(1) 27 15
+ 7 +23



(in ink print, the examples are side-by-side)

(2) 1. 42 2. 100
—23 — 91



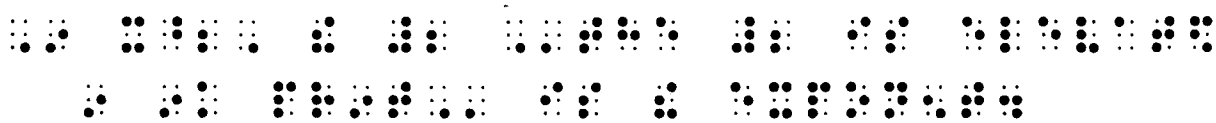
(in ink print, the examples are side-by-side)

§186. Transcriber's Notes:

a. Transcriber's notes must be enclosed by the transcriber's grouping symbols.

b. A transcriber's note consisting of seven words or less may be inserted directly into the text at the point where it applies. Longer notes must be placed at the nearest convenient point relative to the material to which they apply and must be placed, indented, and run over in accordance with the rules of the *Code of Braille Textbook Format and Techniques.*

(1) In x^2, the ^{2} is the exponent.



§187. Keying Technique:

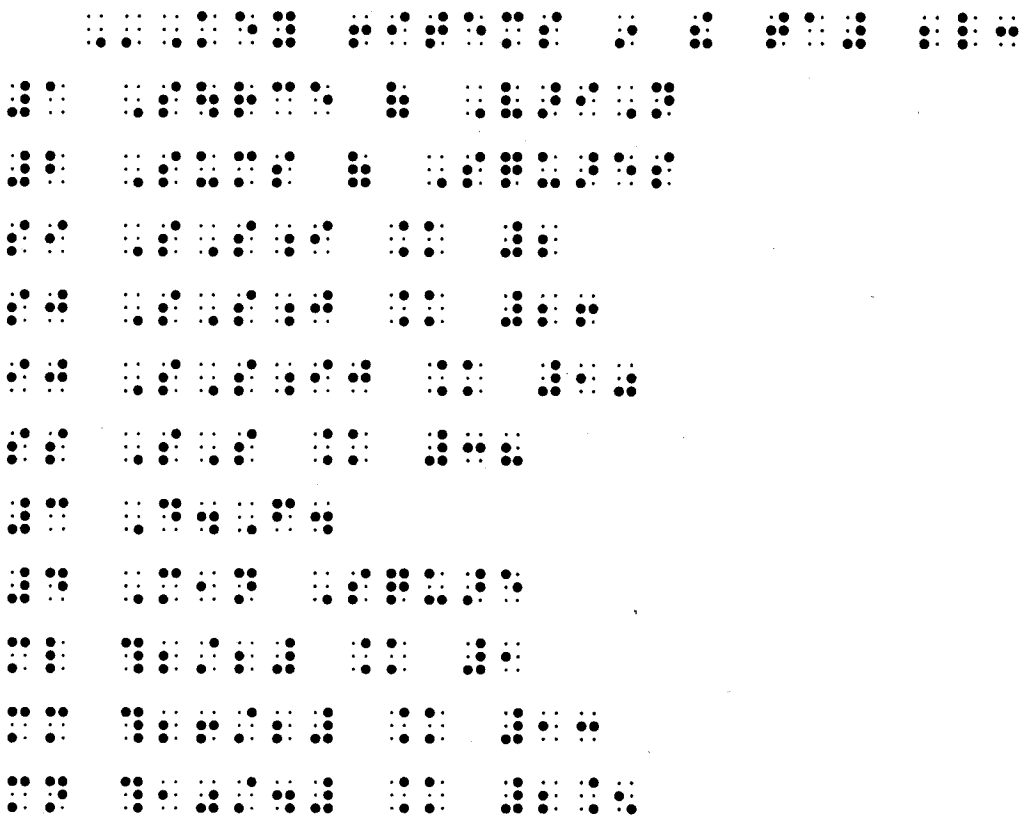
a. When space does not permit the inclusion of labels, column headings, entries, etc., in a figure, determinant, matrix, or table as shown in ink print, one or more of the labels, headings, entries, etc. may be replaced by a numeric or alphabetic key. A numeric key should consist of a numeral written in the upper part of the braille cell. This numeral must be preceded by the numeric indicator and must not be punctuated. An alphabetic key must consist of two lower-case English letters and, if possible, the combination should be suggestive of the item it represents. An alphabetic key may only be used when the author's entries are never composed of two lower-case letters. Two items which are identical should have the same key assigned to them.

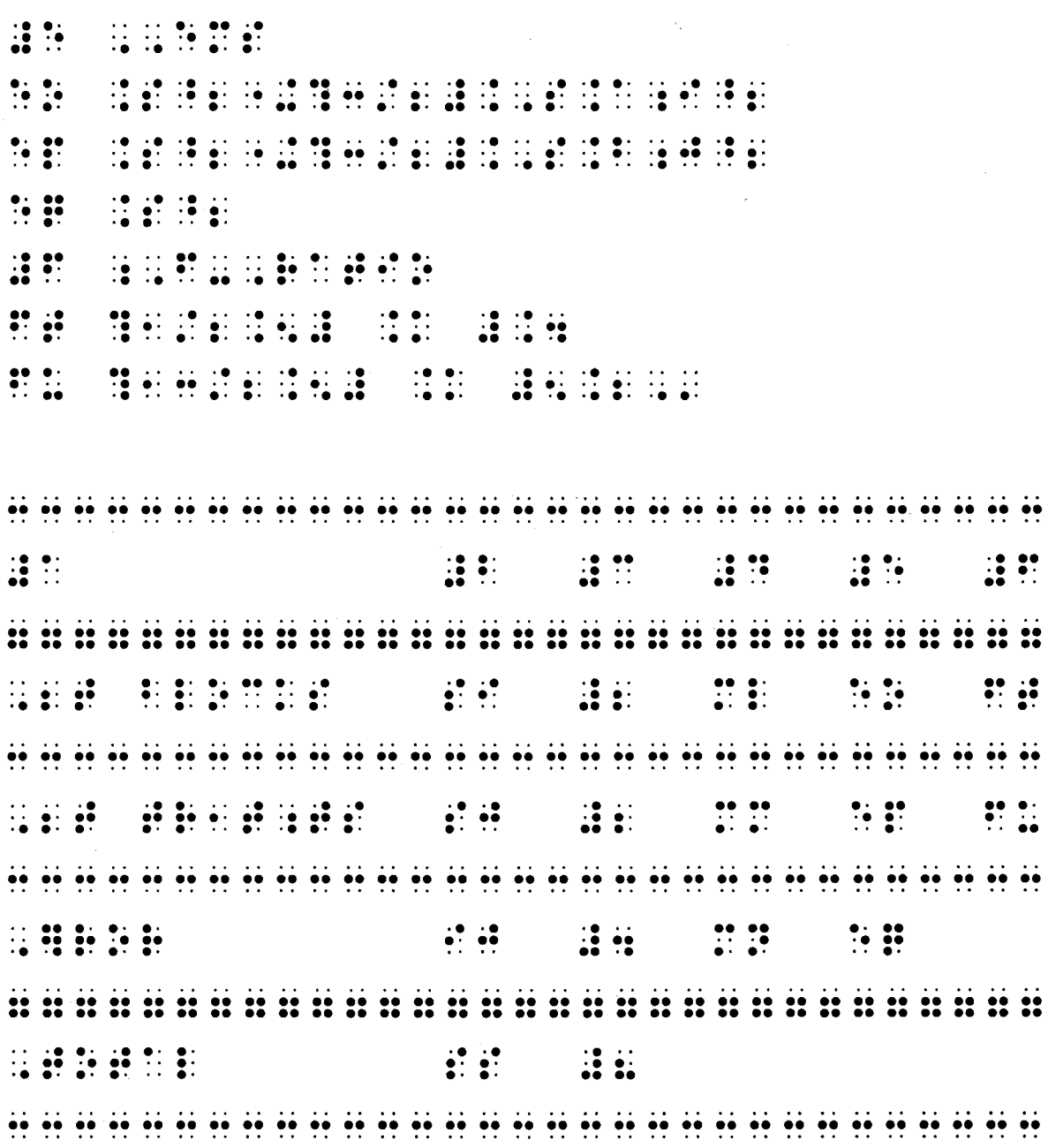
b. If a list of numeric keys is used, it must consist of consecutive numerals beginning with number 1, and these numerals should be placed in the figure, determinant, matrix, or table in the same position as the material which they replace.

c. A list of numeric or alphabetic keys and their meanings must be enclosed in transcriber's grouping symbols and must precede the material to which it applies. Key items may be arranged vertically at the margin, or they may be arranged in columns to save space. A key listing must be preceded and followed by a blank line. If possible, this list must be placed on the same braille page as the material to which it applies.

(1)

Source of Variation	Sums of Squares	D.F.	Mean Square	EMS	F-Ratio
Between blocks	$SS_i = 2$	2	$\frac{2}{2} = 1$	$\sigma^2 + \frac{3}{2}\Sigma\alpha_i^2$	$\frac{1}{2.5} = .4$
Between treatments	$SS_j = 26$	2	$\frac{26}{2} = 13$	$\sigma^2 + \frac{3}{2}\Sigma\beta_j^2$	$\frac{13}{2.5} = 5.2$
Error	$SS_{ij} = 10$	4	$\frac{10}{4} = 2.5$	σ^2	
Total	$SS = 38$	8			





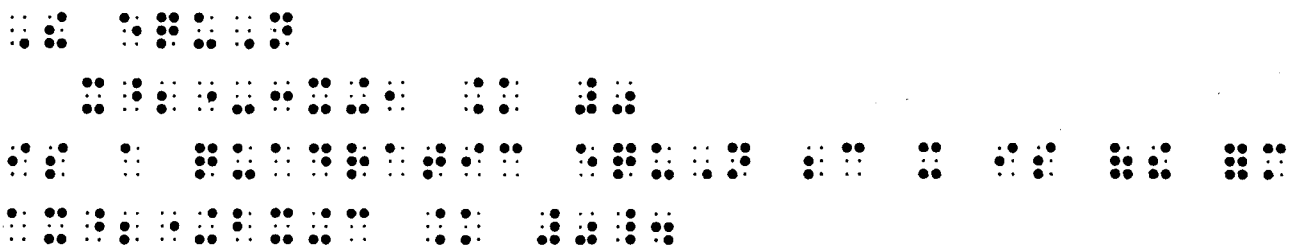
§188. Displayed and Embedded Expressions:

a. Whenever an expression is set apart from the body of the text by skipped lines, indentation, or some other means, it will be referred to as a *displayed expression.* By contrast, an expression which is not set apart from the body of the text by any of the above means will be referred to as an *embedded expression.*

(1) The equation

$$x^2 - 3x + 1 = 0$$

is a quadratic equation because it is of the form $ax^2 + bx + c = 0$.



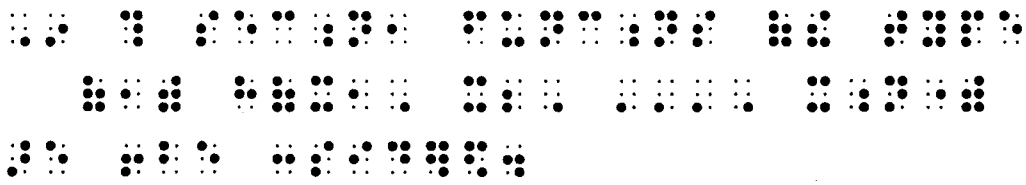
(the first equation is a displayed expression; the second equation is an embedded expression)

b. If an identifying number or letter is associated with a displayed expression, this number or letter is part of the displayed expression and must begin in the appropriate cell in accordance with the rules for displayed expressions in **§§190-191.** In ink print, identifying numbers or letters are sometimes at the right. In braille, numbers or letters must be placed uniformly at the left. However, if identifying numbers or letters occur at the right in ink print, a transcriber's note concerning the transposition of such numbers or letters must be placed at the beginning of the first volume. Page references which are associated with an expression must immediately follow that expression.

(1) In this section, functions of the type

$$h(x_1, x_2, \ldots, x_n) \qquad (1)$$

are to be considered.

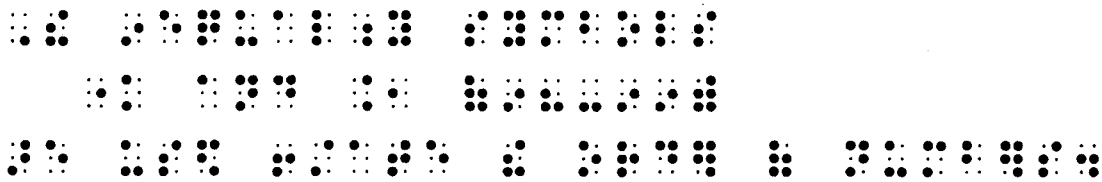


(in ink print, the identifying number occurs at the right)

(2) The inequality symbols

$$< \quad \text{and} \quad > \qquad (98\text{-}99)$$

are used to state the order of numbers.



(in ink print, the page numbers occur at the right-hand margin of the page)

§189. Linked Expressions:

a. A linked expression must contain at least one sign of comparison. The component which precedes the first sign of comparison is called the *anchor*. Each of the remaining components, beginning with a comparison sign but not including the next comparison sign, is called a *link*.

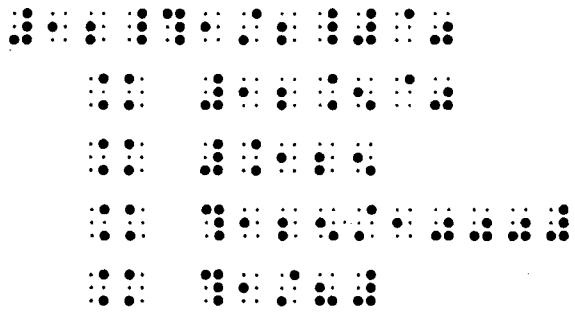
b. When a linked expression meets the following criteria, it is subject to the special margin requirements set forth in **§190c, §191a(iv) and b(v).**

i. The expression must be displayed and not embedded within text.

ii. Its signs of comparison must be vertically aligned in print, except possibly for the last few which may occur on the last print line of the expression.

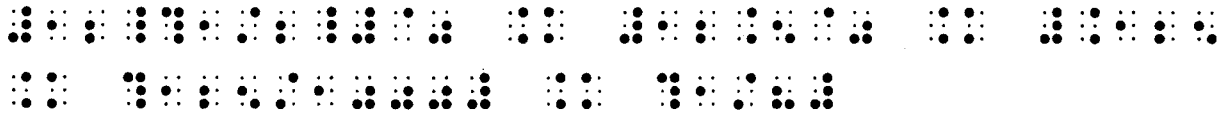
iii. No sign of comparison, except possibly the first one, may be preceded by any expression on its left.

(1) $12\frac{1}{2}\% = 12.5\%$
$= .125$
$= \frac{125}{1000} = \frac{1}{8}$



(in ink print, the first equals sign appears to the right of $12\frac{1}{2}\%$ and all other equals signs except the last one are aligned beneath it)

(2) $12\frac{1}{2}\% = 12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$

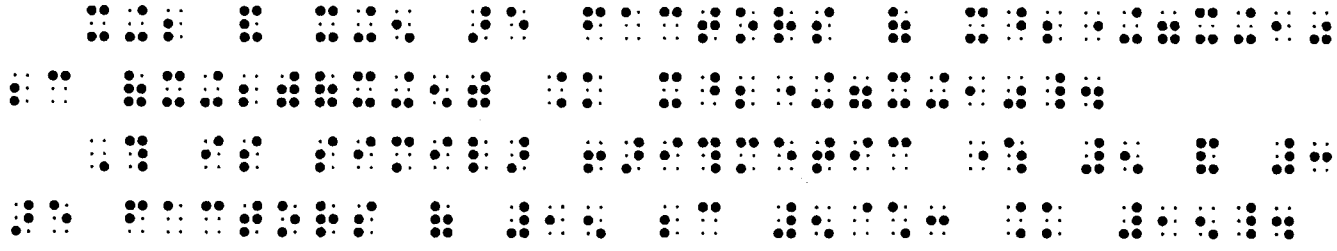


(a linked expression which does not require special margin provisions; in ink print the entire expression appears on one line)

§190. Margins for Narrative Portions of Text:

a. In narrative portions of text, margins should be maintained as in English Braille; paragraphs must begin in cell 3 and must be run over, if necessary, in cell 1.

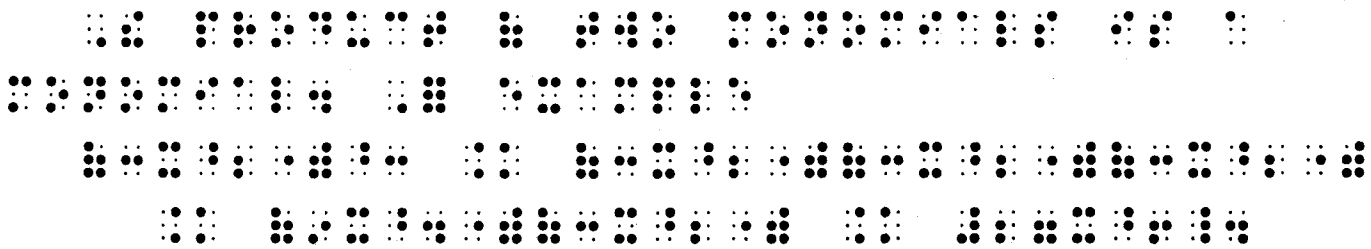
(1) $x + 2$ and $x + 5$ are factors of $x^2 + 7x + 10$ because $(x + 2)(x + 5) = x^2 + 7x + 10$.
This is similar to arithmetic where 5 and 3 are factors of 15 because $5 \times 3 = 15$.



b. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 3 and must be run over, if necessary, in cell 5.

(1) The product of two monomials is a monomial.
For example

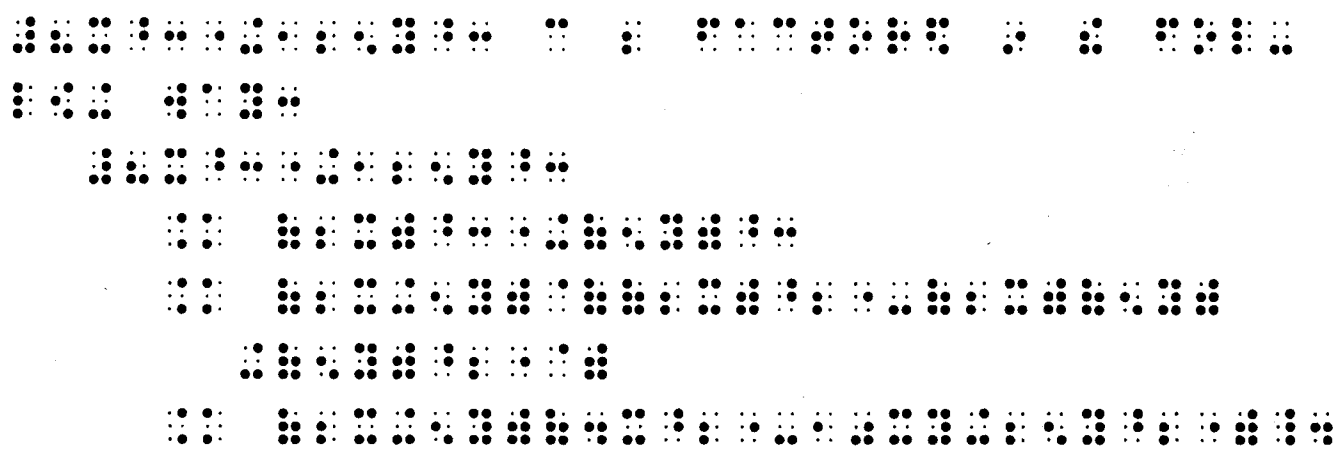
$$(3x^2)^3 = (3x^2)(3x^2)(3x^2) = (9x^4)(3x^2) = 27x^6.$$



c. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 3 and must be run over, if necessary, in cell 7. Each link must begin in cell 5 on a new braille line and must be run over, if necessary, in cell 7.

(1) $8x^3 + 125y^3$ can be factored in the following way:

$$\begin{aligned} 8x^3 + 125y^3 &= (2x)^3 + (5y)^3 \\ &= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2] \\ &= (2x + 5y)(4x^2 - 10xy + 25y^2). \end{aligned}$$



(in ink print, the first equals sign appears to the right of $8x^3 + 125y^3$ and all other equals signs are aligned beneath it)

§191. **Margins for Non-Spatial Itemized Materials:** When material is identified sequentially by number or letter, as in exercises or outlines, it will be referred to as *itemized material.*

a. When non-spatial itemized material contains main divisions only (no subdivisions) the following rules concerning margins must be observed:

i. The main division numbers or letters must begin in cell 1 and the associated material must be run over, if necessary, in cell 3.

ii. Succeeding paragraphs, if any, must begin in cell 5 and must be run over, if necessary, in cell 3.

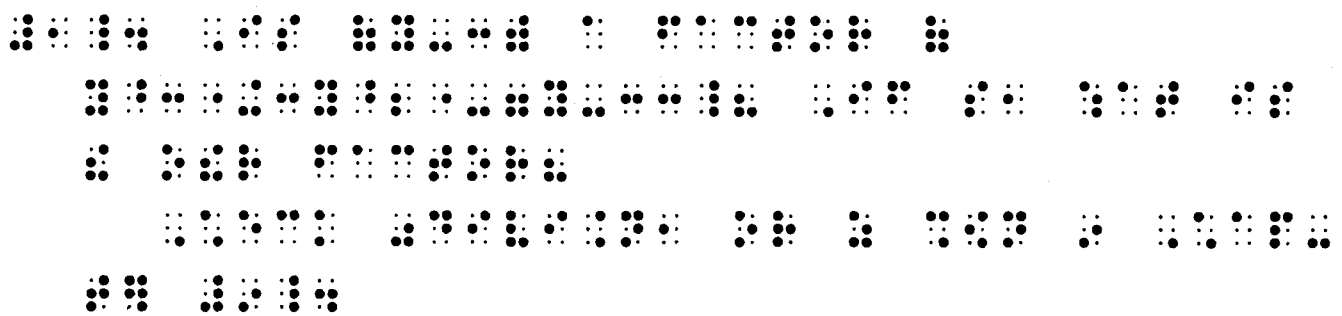
iii. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 5 and must be run over, if necesary, in cell 7.

iv. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 5 and must be run over, if necessary, in cell 9. Each link must begin in cell 7 and must be run over, if necessary, in cell 9.

v. Instructions which apply to a group of problems which follow must begin in cell 5 and must be run over, if necessary, in cell 3. There must be a blank line above such instructions, but not below. However, a page-change line may take the place of this required skipped line. The last line of an instruction and the first line of a problem to which it applies must be on the same braille page.

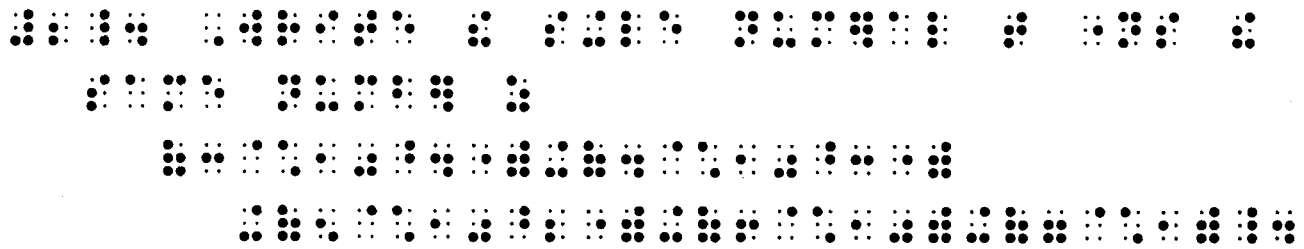
(1) 1. Is $(y - 3)$ a factor of $y^3 + 3y^2 - 7y - 33$? If so, what is the other factor?

Check by division, or as shown in Chapter 9.



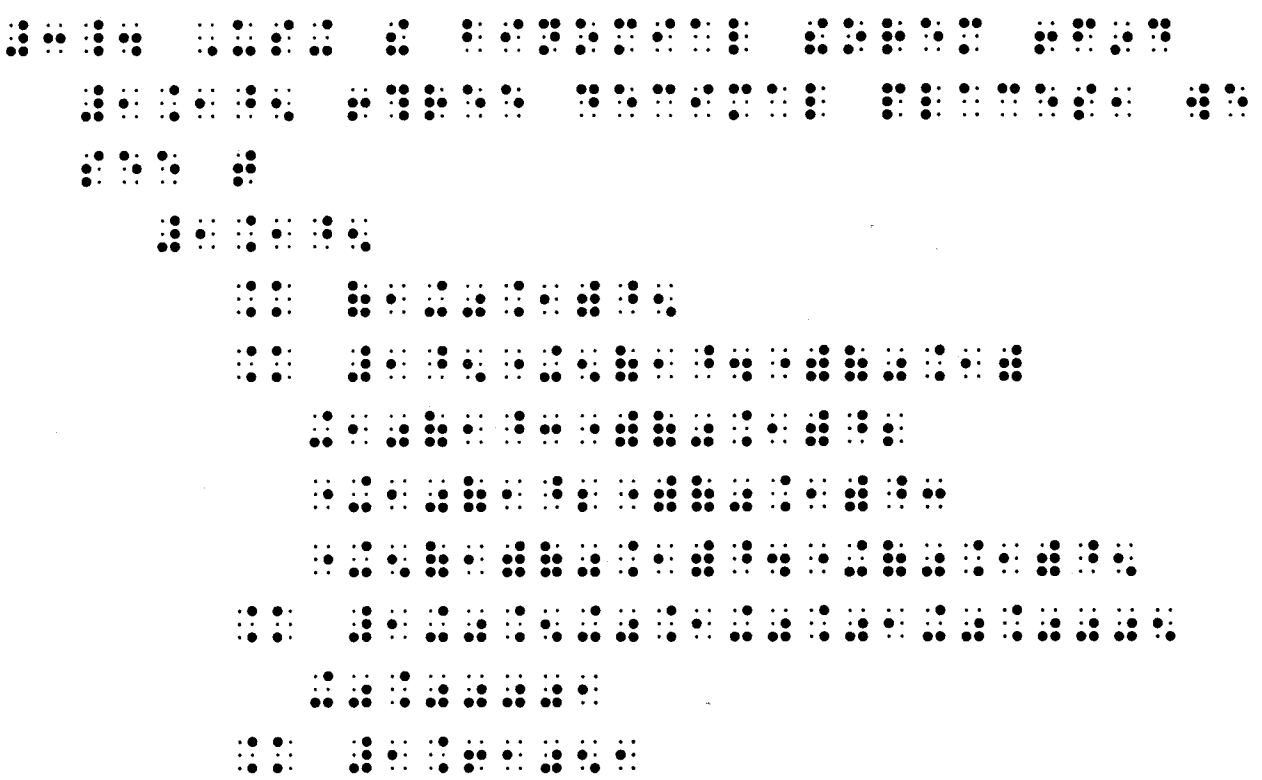
(2) 2. Write the single numeral that names the same number as

$$(3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) + (6 \times 10) + (7 \times 1).$$



(3) 3. Using the binomial theorem to find 1.1^5 to three decimal places, we see that

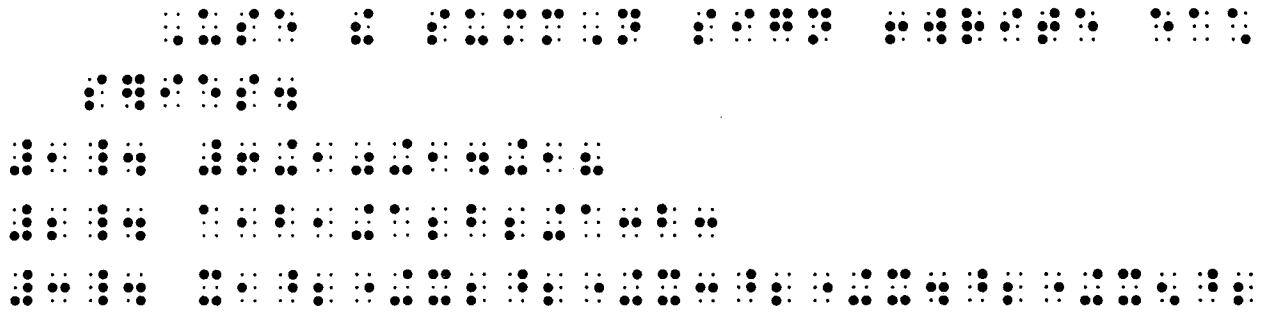
$$\begin{aligned} 1.1^5 &= (1 + 0.1)^5 \\ &= 1^5 + 5(1^4)(0.1) + 10(1^3)(0.1)^2 + 10(1^2)(0.1)^3 + \\ &\qquad 5(1)(0.1)^4 + (0.1)^5 \\ &= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001 \\ &= 1.61051 \end{aligned}$$



(in ink print, the first equals sign is to the right of 1.1^5 and all other equals signs are aligned beneath it)

(4) Use the summation sign to write each series.

1. $6 + 10 + 14 + 18$
2. $a_1b_1 + a_2b_2 + a_3b_3$
3. $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$



b. When non-spatial itemized material contains both main divisions and subdivisions to whatever depth, the following rules concerning margins must be observed:

i. The main division numbers or letters must begin in cell 1 and the associated material must be run over, if necessary, in cell 5.

ii. Subdivision numbers or letters, regardless of depth, must begin in cell 3 and must be run over, if necessary, in cell 5.

iii. Succeeding paragraphs, if any, must begin in cell 7 and must be run over, if necessary, in cell 5.

iv. When the special margin requirements for linked expressions do not apply, a displayed expression must begin in cell 7 and must be run over, if necessary, in cell 9.

v. When the special margin requirements for linked expressions do apply, the anchor must begin in cell 7 and must be run over, if necessary, in cell 11. Each link must begin in cell 9 and must be run over, if necessary, in cell 11.

vi. Instructions which apply to a group of problems which follow must begin in cell 5 and must be run over, if necessary, in cell 3. There must be a blank line above such instructions, but not below. However, a page-change line may take the place of this required skipped line. The last line of an instruction and the first line of a problem to which it applies must be on the same braille page.

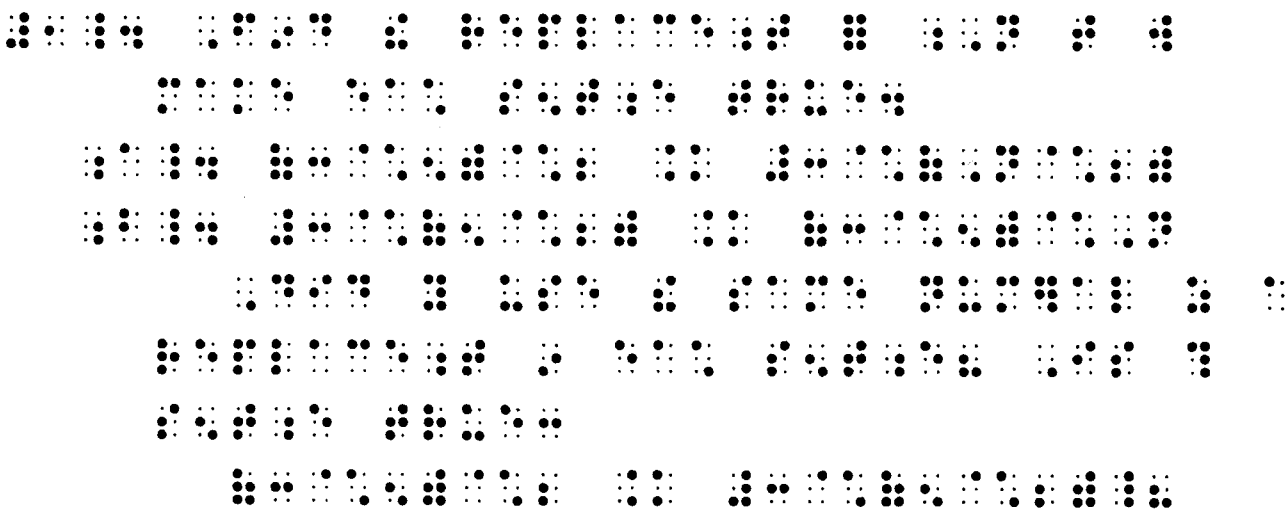
(1) 1. Find the replacement for N that will make each sentence true.

a. $(3 \times 5) \times 2 = 3 \times (N \times 2)$

b. $3 \times (5 \times 2) = (3 \times 5) \times N$

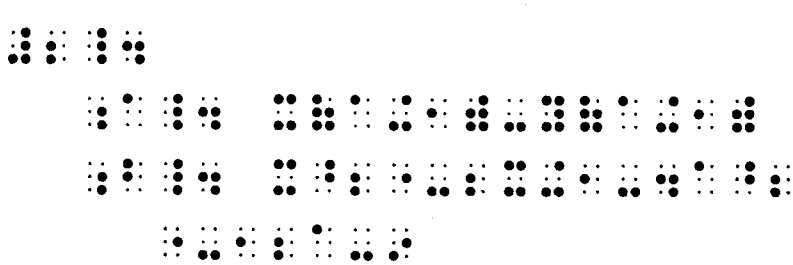
Did you use the same numeral as a replacement in each sentence? Is this sentence true:

$(3 \times 5) \times 2 = 3 \times (5 \times 2)$?



(2) 2. a. $x(a + 1) - y(a + 1)$

b. $x^2 - 2x + 1 - 4a^2 - 12a - 9$



(in ink print, the a is on the same line as the problem number, and the b is aligned beneath a)

(3) 3. In factoring $ab + c^2 + ac + bc$:

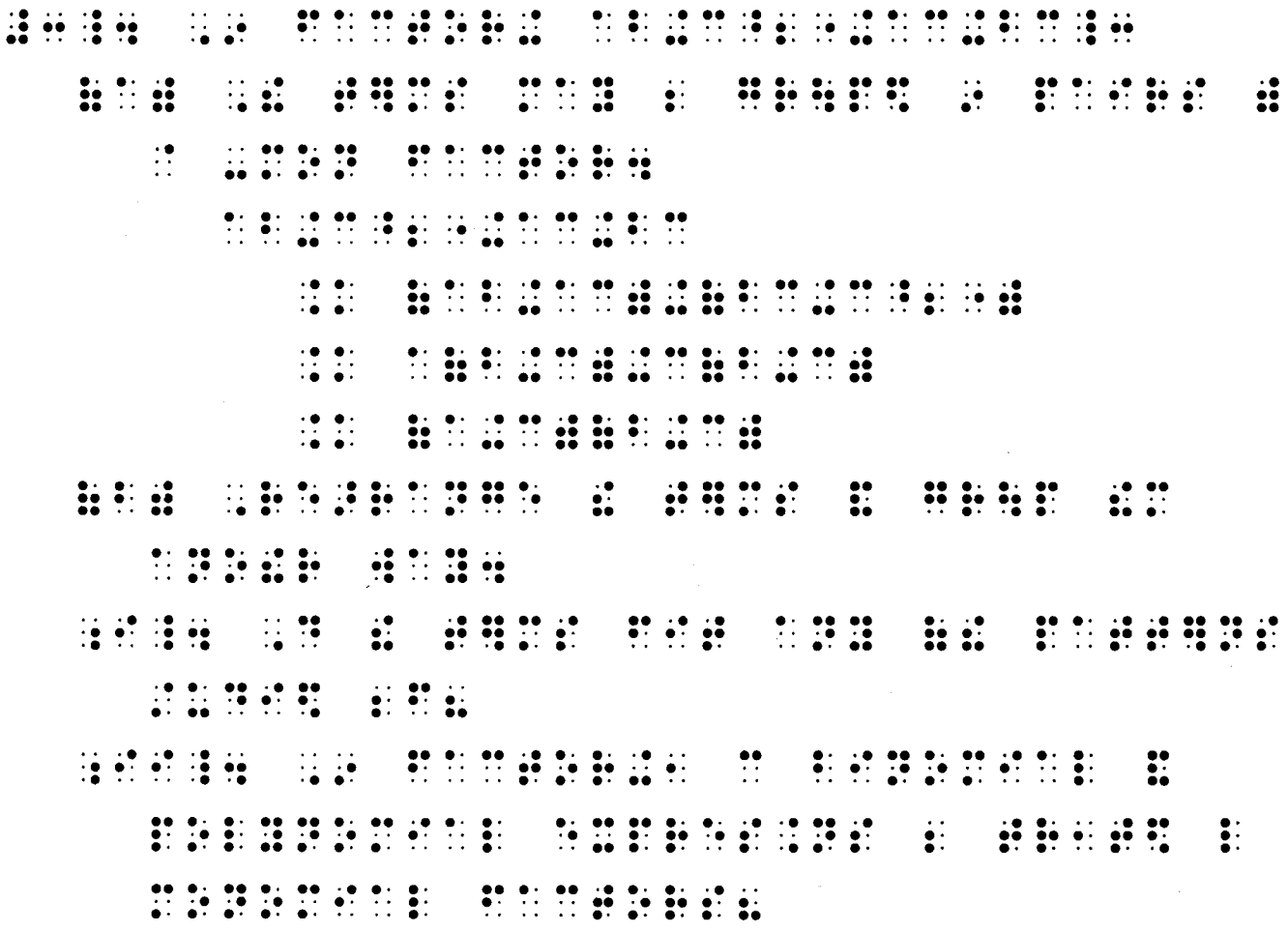
(a) The terms may be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c) \end{aligned}$$

(b) Rearrange the terms and group them another way.

i. Do the terms fit any of the patterns studied before?

ii. In factoring, can binomial and polynomial expressions be treated like monomial factors?

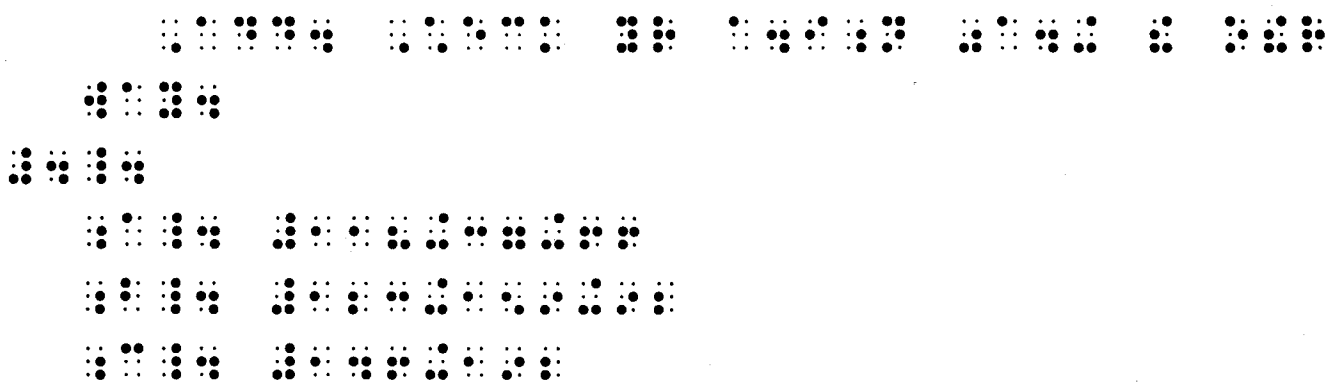


(in ink print, the main problem number, the (a) and the (b) are vertically aligned; the Roman numerals are indented further to the right)

(4) Add. Check your addition by adding the other way.

4. a. 118 + 37 + 66 b. 123 + 159 + 92

c. 146 + 192

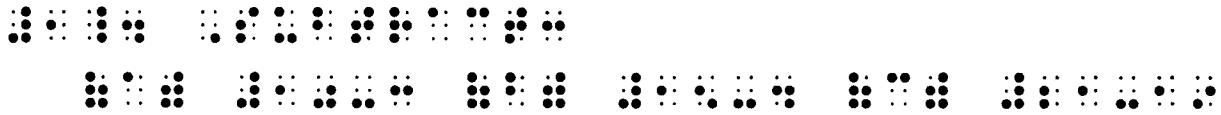


(in ink print, the problem number and the first two subdivisions are on the same line; the third subdivision is beneath the first)

c. When non-spatial itemized material contains both main divisions and subdivisions, it is permissible to place all subdivisions on a single braille line if that braille line can accommodate all the subdivisions.

(1) 1. Subtract:

(a) 10 — 3 (b) 15 — 4 (c) 21 — 19



(the braille and ink print are the same)

§192. Margins for Spatial Itemized Materials: Spatial itemized material may be transcribed using the same margin rules as are contained in §191. However, for space-saving purposes the following alternatives are available:

a. When spatial itemized material contains main divisions only (no subdivisions), the first division number begins in cell 1. Subsequent division numbers may begin to the right of the preceding spatial arrangement regardless of how they occur in ink print. As many main division numbers and their associated spatial arrangements may occur across the page as can be accommodated. If additional main division numbers remain, the first of these begins again in cell 1, after having left a blank line below the longest of the spatial arrangements which occur above.

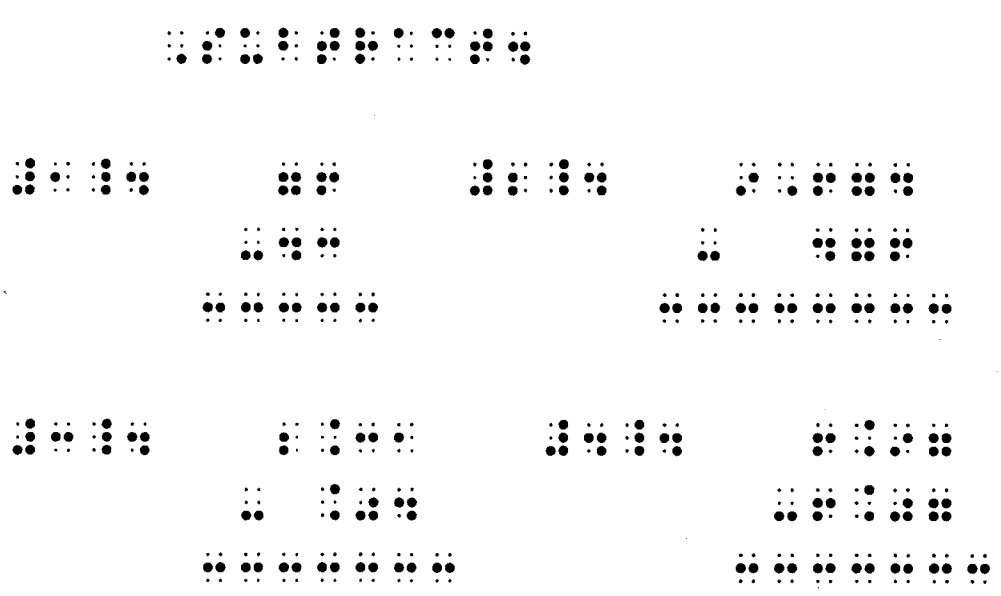
(1) Subtract.

1. 76
   — 43

2. 9,674
   — 476

3. 2.31
   — .04

4. 6.97
   — 6.07

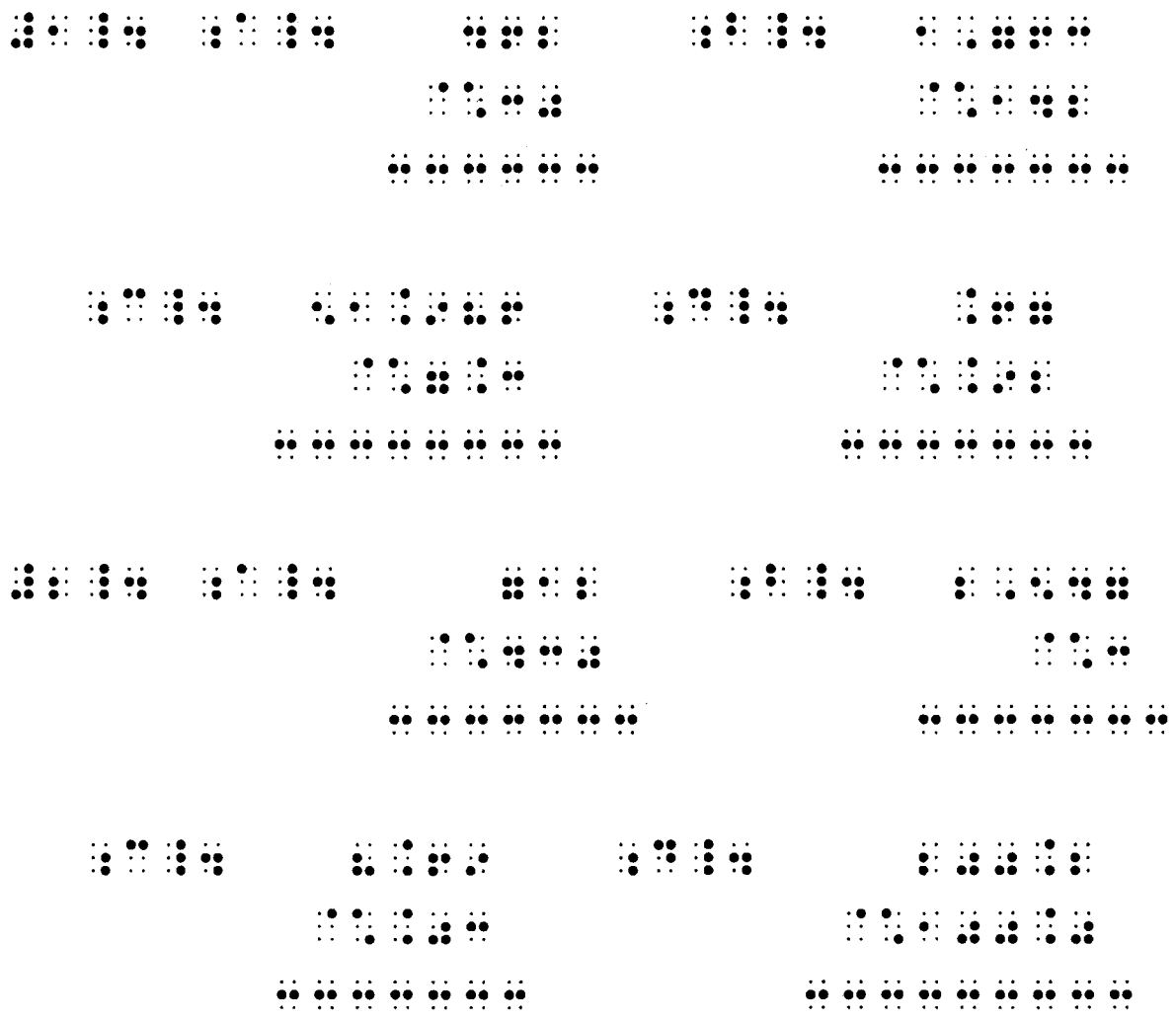


(in ink print, examples 1 and 2 appear in the first column, examples 3 and 4 appear in the second column)

b. When spatial itemized materials contain both main divisions and subdivisions, the first main division number begins in cell 1 and the first subdivision follows on the same braille line if there is no material between the main division number and the subdivison number. As many additional subdivisions may be transcribed across the line as can be accommodated. If additional subdivisions remain, they are started in cell 3 after having left a blank line below the longest of the spatial arrangements which occur above.

(1) 1. a. 462 ×30 b. 1,763 × 142 c. 51.986 × 7.3 d. .67 ×.92

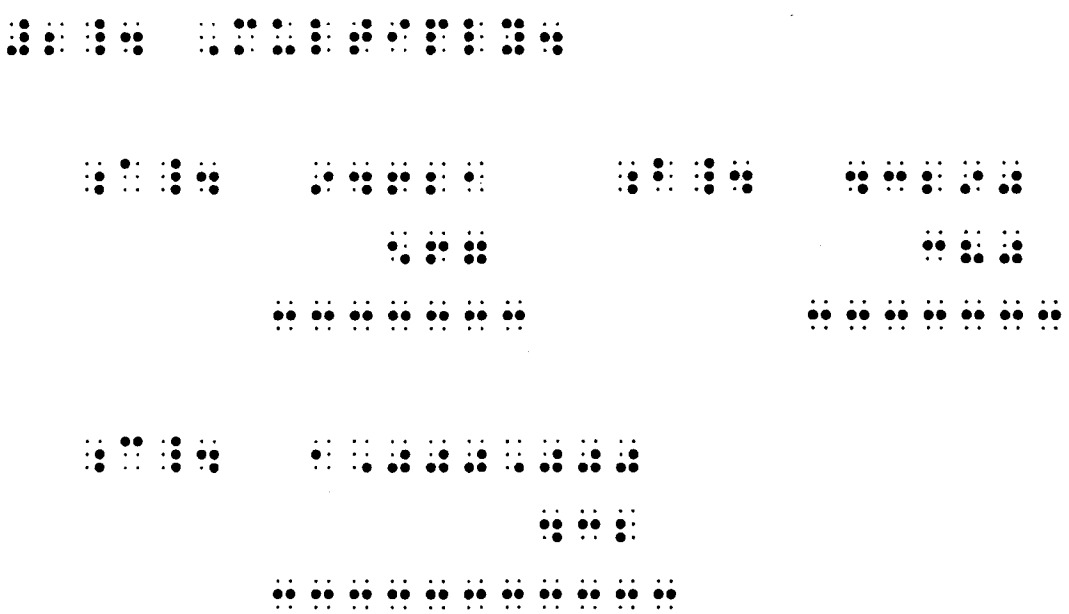
2. a. 712 ×430 b. 2,547 × 3 c. 8.69 × .03 d. 200.2 ×100.0



(in ink print, all four subdivisions of each problem appear across the page in columnar form)

(2) 2. Multiply.

a. 94621 b. 43290 c. 1,000,000
567 380 432



(in ink print, all the subdivisions are on the same line)

§193. Margins for Spatial and Non-Spatial Itemized Materials Arranged in Tabular Form:

a. When itemized material is arranged in tabular form so that rows are identified by number and columns are identified by letter, the following technique must be used provided that the *entire* tabulation can be contained across the braille page.

i. The letters which identify the columns must be left-justified in the columns to which they apply.

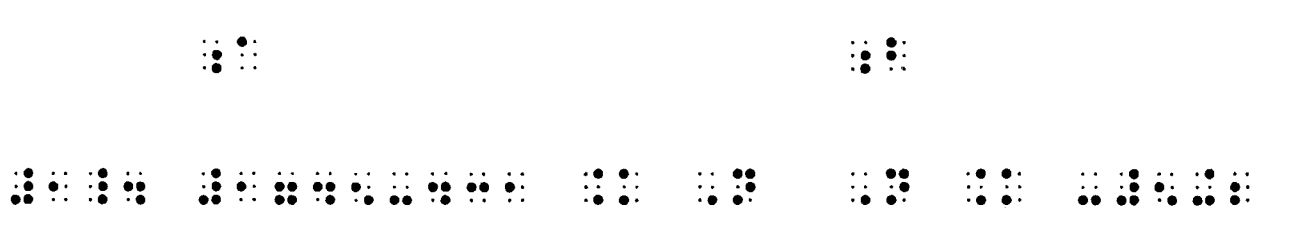
ii. A blank line must be left above and below the column headings.

iii. Row numbers must begin in cell 1.

iv. At least two spaces should be left between the right-hand margin of one column and the left-hand margin of the next column.

(1)

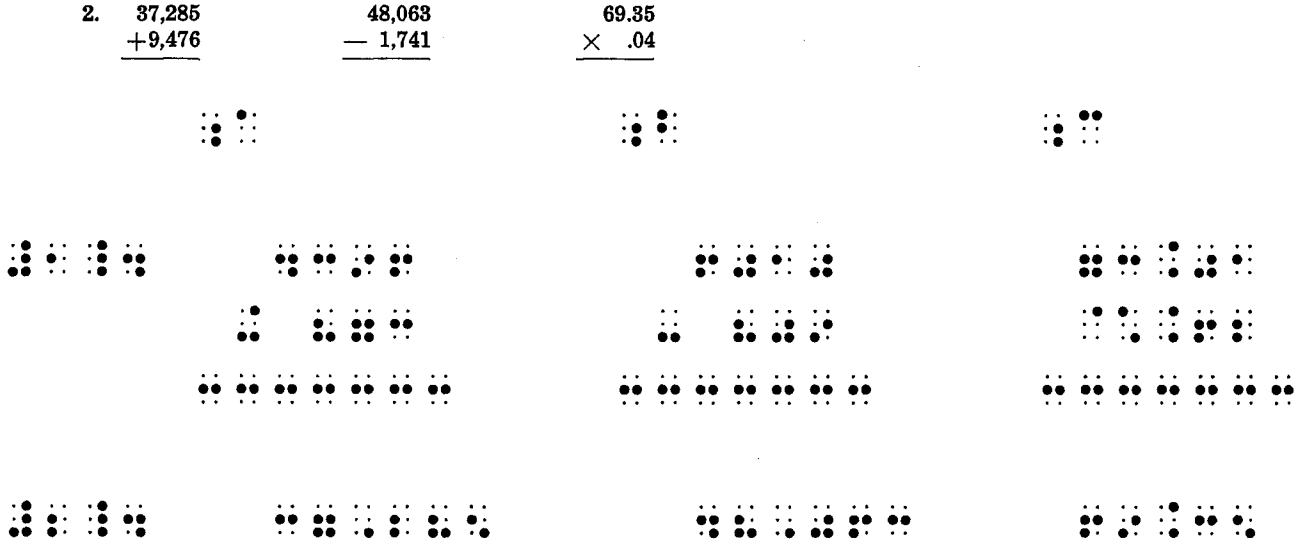
	a	b
1.	$1745 - 431 = N$	$N = -5 + 2$
2.	$N = 18 + (-9)$	$-7 + 14 = N$



(ink print and braille are the same)

(2)

	a	b	c
1.	4396 + 873	6010 — 809	73.01 × .62
2.	37,285 +9,476	48,063 — 1,741	69.35 × .04

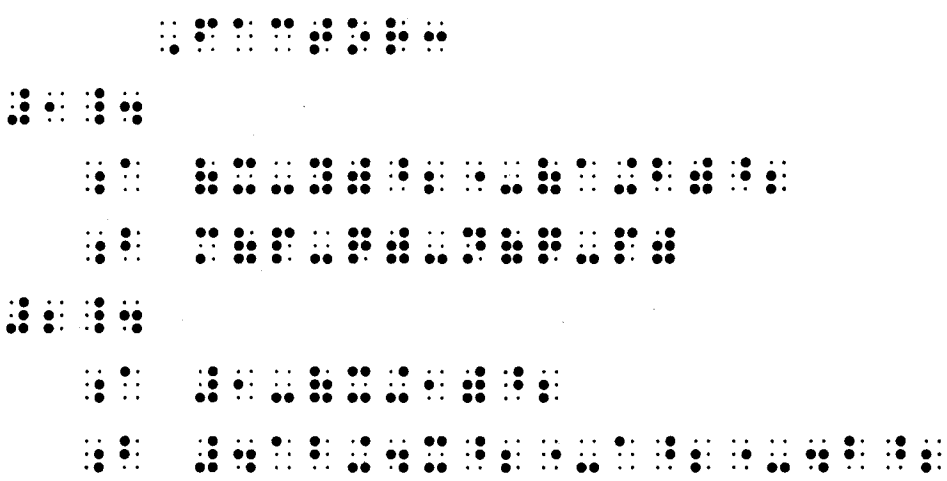


(ink print and braille are the same)

b. If the entire tabulation cannot be contained across the braille page using the technique of **a** above, the transcription should proceed as if the row numbers were main headings and the column letters were subdivision headings for each main division number. In such cases, the margin rules of §191b and §192b then apply.

(1) Factor:

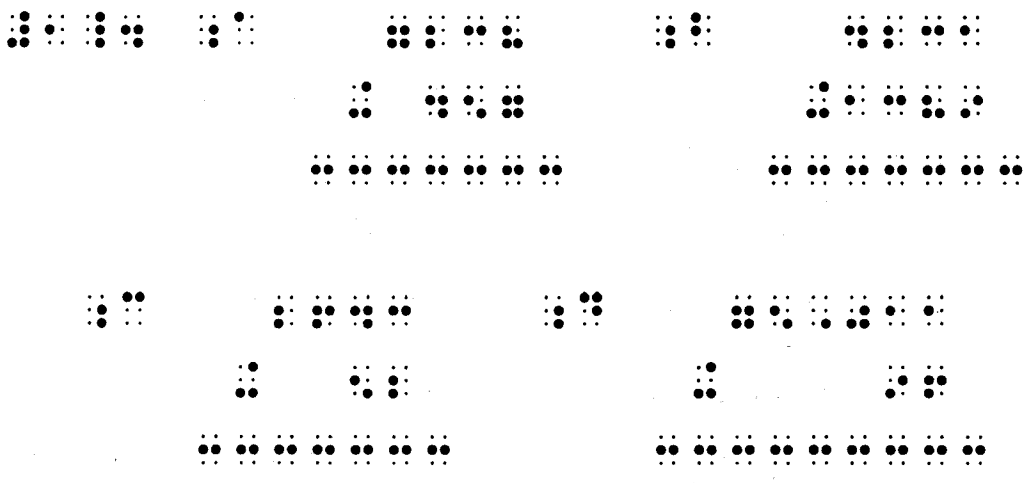
	a	b
1.	$(x-y)^2-(a+b)^2$	$m(p-q)-n(q-p)$
2.	$1-(x+1)^2$	$4ab+4x^2-a^2-4b^2$



(in ink print, the subdivisions are placed across the page and are aligned beneath lettered column headings)

(2)

	a	b	c	d
1.	7238	4231	2643	75,011
	+ 457	+1389	+ 52	+ 96



(in ink print, the subdivisions are placed across the page and are aligned beneath lettered column headings)

§194. Format for Formal Proofs:

a. A formal proof is usually introduced by a word such as *Theorem, Proposition,* or *Lemma.* The following format is recommended for the transcription of such formal proofs:

i. A line must be skipped before the beginning of the formal proof.

ii. The fully capitalized word *Theorem, Proposition,* or *Lemma* must begin in cell 8 and the statement following this word should be run over, if necessary to cell 1.

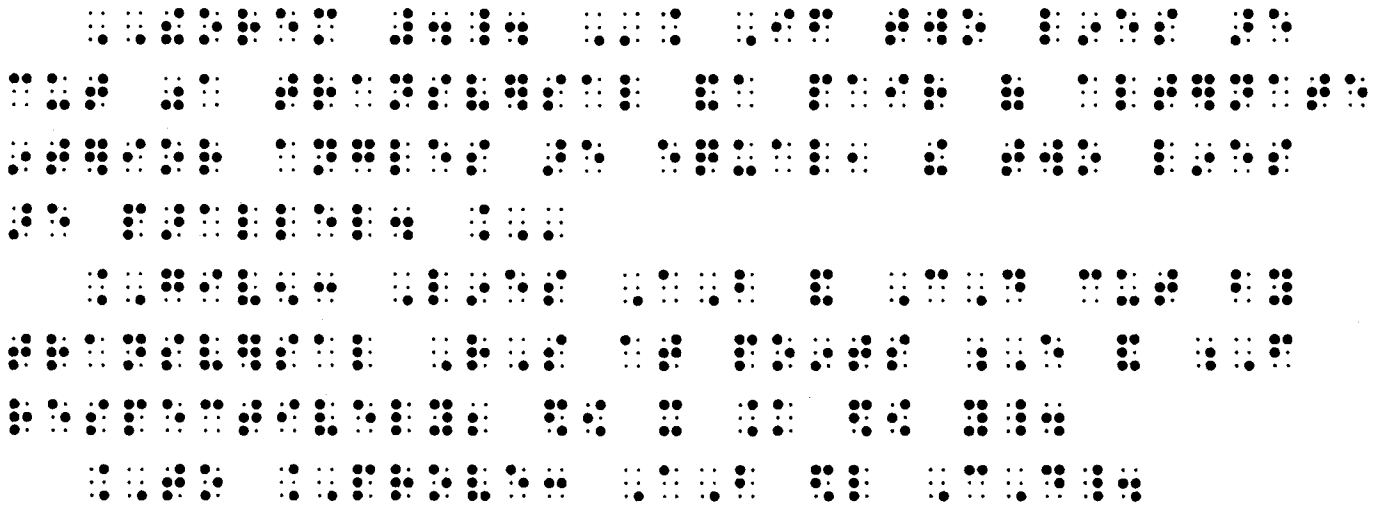
iii. Auxiliary captions such as *Given, Hypothesis, Prove,* or *Conclusion* must follow, without a skipped line, and must begin in cell 3. These captions should be capitalized or italicized in accordance with the print text. If they are in boldface type, they should be written as fully capitalized in braille. Material associated with these captions should follow the captions and should be run over, if necessary, to cell 1. However, when a proof is presented by step number, a line should be left blank after the caption *proof* and the format in b below must be followed.

iv. When the formal proof is complete, a line must be left blank before continuing with the text.

(1) **Theorem 4.** *If two lines are cut by a transversal and a pair of alternate interior angles are equal, the two lines are parallel.*

Given: Lines AB and CD cut by transversal RS at points E and F respectively; $\angle x = \angle y$.

To Prove: AB $\parallel$ CD.



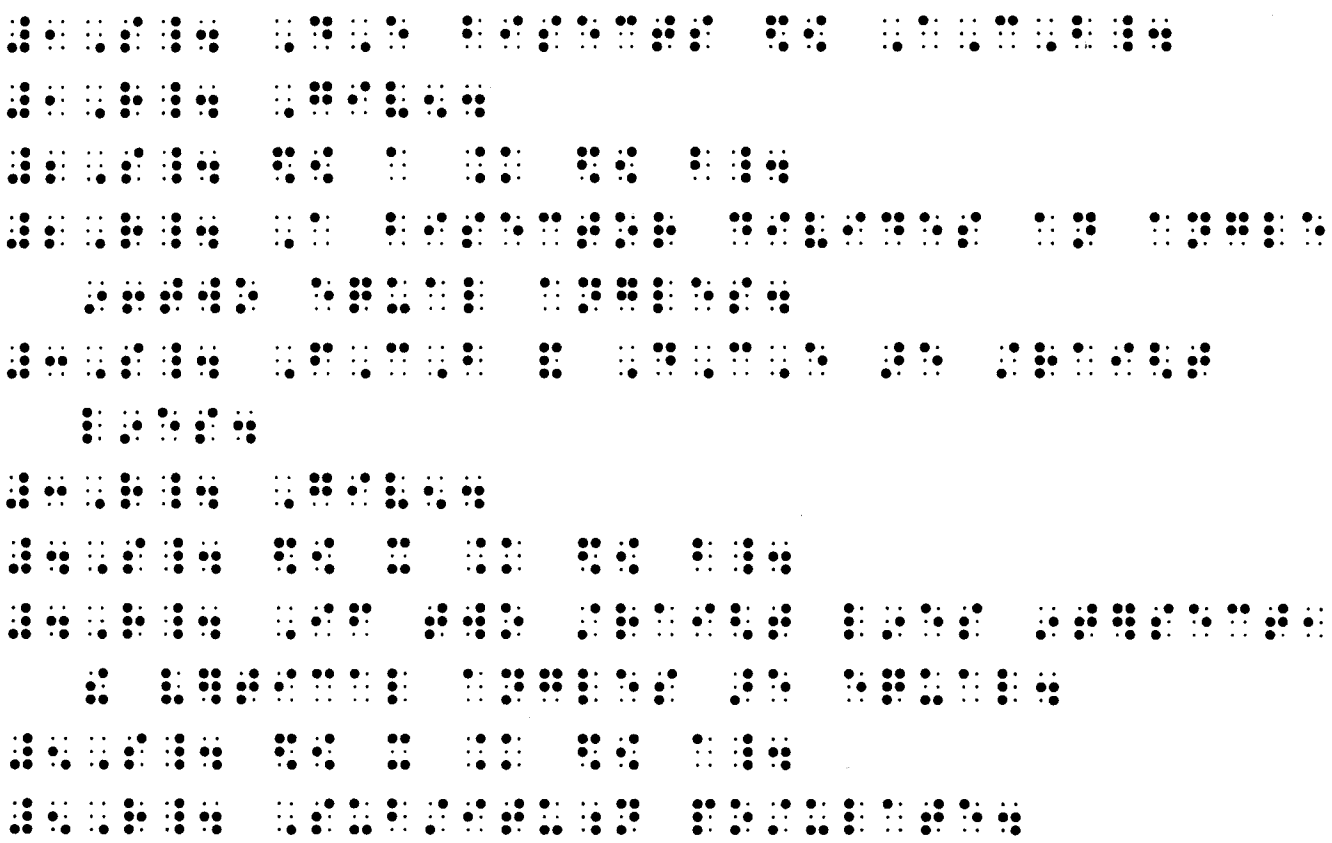
b. When a formal proof is presented by step number and is divided into two columns headed "Statement" and "Reason", the following technique must be used:

i. All step numbers must begin at the margin.

ii. The step number must be followed by the letter "S" or "R" according as the transcription to follow is from the Statement or the Reason column. The transcription must begin on the same line as the step number and runovers, if necessary, must begin in cell 3. If a caption other than "Statement" or "Reason" is used, a suitable letter should be used for "S" or "R".

iii. A transcriber's note must be included to call attention to this braille format and to specify the meaning of "S", "R", or other letters which may have been used. This note must be placed at the beginning of each braille volume in which this technique is used.

(1)	*Statements*	*Reasons*
1.	DE bisects $\angle ACB$.	1. Given.
2.	$\angle a = \angle b$.	2. A bisector divides an angle into two equal angles.
3.	FCB and DCE are straight lines.	3. Given.
4.	$\angle x = \angle b$.	4. If two straight lines intersect, the vertical angles are equal.
5.	$\angle x = \angle a$.	5. Substitution postulate.

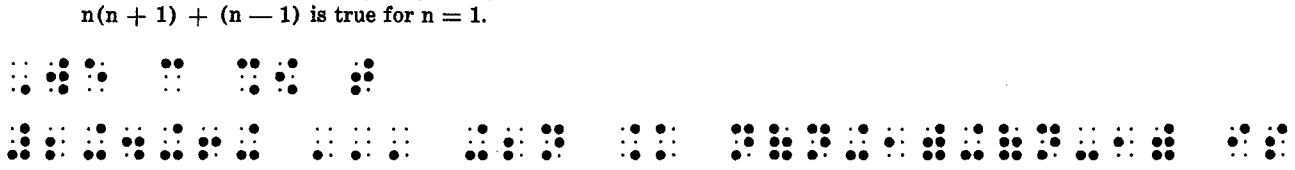


(in ink print, there is a Statement column and a Reason column)

§195. Runovers: The runover of a mathematical expression to another braille line must be avoided subject to the margin requirements which are in effect at the time of transcribing the expression.

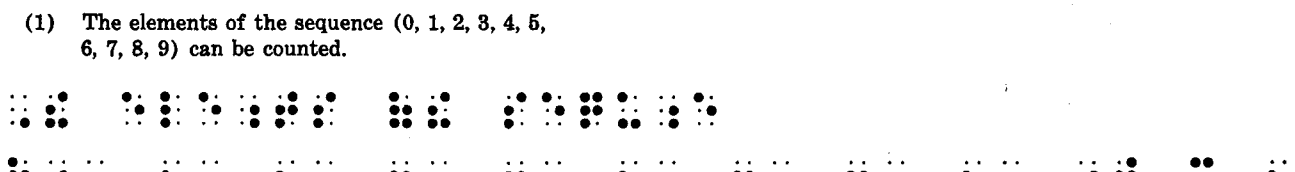
a. As much of a braille line must be left blank as necessary in order to keep all of a mathematical expression on a single braille line.

(1) We can show that $2 + 4 + 6 + \ldots + 2n = n(n + 1) + (n - 1)$ is true for $n = 1$.



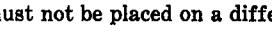
b. A sequence of mathematical expressions which occurs in an "enclosed list" must not be divided between braille lines if all of the "enclosed list" can be kept on a single braille line.

(1) The elements of the sequence (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) can be counted.

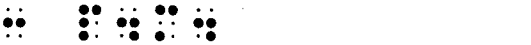


c. An abbreviation must not be placed on a different braille line from its preceding or following numeral or letter.

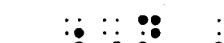
(1) 4 in

(2) 3 p.m. 

(3) 30° C 

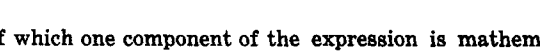
(4) N 30° W 

(5) Fig. 6.10 

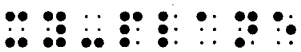
(6) x ft. 

d. A hyphenated expression of which one component of the expression is mathematical must not be divided between braille lines.

(1) 6-inch ruler 

(2) 4-sided figure 

(3) x-intercept 

(4) xy-plane 

e. When a mathematical expression cannot be kept on one braille line and must be divided between lines, the division must be made giving priority to the following items in descending order:

i. After a comma which occurs between items in an "enclosed list."

ii. Before a symbol of comparison.

iii. Before a symbol of operation.

iv. Before a fraction line.

v. Before the base-line indicator.

vi. Before a change-of-level indicator or within a superscript or subscript before one of the symbols listed above.

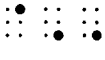
vii. Between factors which are enclosed within grouping symbols.

viii. After a termination indicator.

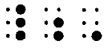
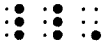
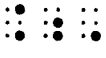
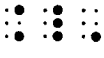
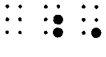
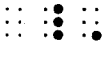
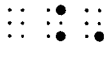
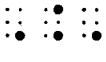
APPENDIX A

COMBINATIONS OF TYPE-FORM, ALPHABETIC, AND CAPITALIZATION INDICATORS

LOWER-CASE LETTERS

Type-form	English letters	German letters	Greek letters	Greek Letter Alternative	Hebrew letters	Russian letters
Boldface						
Italic						
Ordinary						
Sanserif						
Script						

CAPITALIZED LETTERS

Type-form						
Boldface						
Italic						
Ordinary						
Sanserif						
Script						

APPENDIX B

INDEX OF BRAILLE SYMBOLS

The following is the list of 63 braille symbols arranged in their standard order. The separation of these symbols into the usual seven lines of braille is ignored, but each symbol is numbered in accordance with its rank in the list.

1		14		27		40		53	
2		15		28		41		54	
3		16		29		42		55	
4		17		30		43		56	
5		18		31		44		57	
6		19		32		45		58	
7		20		33		46		59	
8		21		34		47		60	
9		22		35		48		61	
10		23		36		49		62	
11		24		37		50		63	
12		25		38		51			
13		26		39		52			

The items in the INDEX OF BRAILLE SYMBOLS are "alphabetized" in accordance with the list of the 63 braille symbols above.

1	⠁ (dot 1)		Page
	⠁	English a	22
		German ah	23
		Greek alpha	23
		Hebrew aleph	24
		Russian ah	25
	⠁⠍⠏	amp (amplitude)	118
	⠁⠝⠞⠊⠇⠕⠛	antilog (antilogarithm)	118
	⠁⠗⠉	arc (arc)	119
	⠁⠗⠛	arg (argument)	119
2	⠃ (dots 1-2)		
	⠃	English b	22
		German beh	23
		Greek beta	23
		Russian beh	25
3	⠉ (dots 1-4)		
	⠉	English c	22
		German tseh	23
		Greek sampi	23
		Russian tseh	25

3 ⠒ (Cont.)

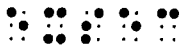
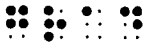
		Page
⠒⠕⠊⠴	colog (cologarithm)	119
⠒⠉⠕	cos (cosine)	119
⠒⠉⠕⠓	cosh (hyperbolic cosine)	119
⠒⠉⠞	cot (cotangent)	119
⠒⠉⠞⠓	coth (hyperbolic cotangent)	119
⠒⠉⠕⠧⠎	covers (coversine)	119
⠒⠉⠎	csc (cosecant)	119
⠒⠉⠎⠓	csch (hyperbolic cosecant)	119
⠒⠉⠞⠝	ctn (cotangent)	119
⠒⠉⠞⠝⠓	ctnh (hyperbolic cotangent)	119

4 ⠲ (dots 1-4-5)

⠲	English d	22
	German deh	23
	Greek delta	23
	Russian deh	25
⠲⠕⠙	det (determinant)	119

5 ⠢ (dots 1-5)

⠢	English e	22
	German eh	23
	Greek epsilon	23
	Russian yeh	25

5	 (Cont.)		Page
		erf (error function)	119
		exp (exponential)	119
		exsec (exsecant)	119
6	 (dots 1-2-4)		
		English f	22
		German eff	23
		Greek phi	24
		Hebrew feh	24
		Russian eff	25
7	 (dots 1-2-4-5)		
		English g	22
		German gheh	23
		Greek gamma	23
		Hebrew gimel	24
		Russian gheh	25
		grad (gradient)	119
8	 (dots 1-2-5)		
		English h	22
		German hah	23
		Hebrew heh	24

8 ⠨ (Cont.)

Page

Russian khah 25

⠨⠓ hav (haversine) 119

9 ⠈ (dots 2-4)

⠈ English i 22

German ee 23

Greek iota 24

Russian ee 25

⠈⠊ im (imaginary part) 119

⠈⠍⠑ inf (infimum) 119

10 ⠘ (dots 2-4-5)

⠘ English j 22

German yaht 23

Hebrew yod 24

Russian zheh 25

11 ⠐ (dots 1-3)

⠐ English k 22

German kah 23

Greek kappa 24

Russian kah 25

12	 (dots 1-2-3)		Page
		English l	22
		German ell	23
		Greek lambda	24
		Hebrew lamed	24
		Russian ell	25
		lim (limit)	119
		ln (natural logarithm)	120
		log (logarithm)	120
13	 (dots 1-3-4)		
		English m	22
		German em	23
		Greek mu	24
		Hebrew mem	24
		Russian em	25
		max (maximum)	120
		min (minimum)	120
		mod (modulo)	120
14	 (dots 1-3-4-5)		
		English n	22
		German en	23

14 ⠞ (Cont.)

Page

Greek nu 24

Hebrew nun 24

Russian en 25

15 ⠇ (dots 1-3-5)

⠗ barbed right full arrowhead > 146

English o 22

German oh 23

Greek omicron 24

Russian oh 25

⠷⠷⠷ curved division sign on left, separation line above ⎫‾‾ 160

⠗

⠇ curved division sign on left, separation line below ⎭___ 161

⠷⠷⠷

⠇ ⠇ curved division signs on left and right, separation line below ⎭___⎩ 161

⠷⠷⠷

16 ⠏ (dots 1-2-3-4)

⠯ English p 22

German peh 23

16	⠿	(Cont.)		Page
			Greek pi	24
			Russian peh	25
17	⠿	(dots 1-2-3-4-5)		
	⠿		English q	22
			German koo	23
			Greek koph (or qoph)	26
			Hebrew koph	24
			Russian cheh	25
18	⠗	(dots 1-2-3-5)		
	⠗		English r	22
			German err	23
			Greek rho	24
			Hebrew resh	25
			Russian err	25
	⠗ ⠗		re (real part)	120
19	⠎	(dots 2-3-4)		
	⠎		English s	22
			German ess	23
			Greek sigma	24
			Hebrew samekh	24

19 ⠭ (Cont.)

	Page
Russian ess	25
⠭⠎ sec (secant)	120
⠭⠎⠓ sech (hyperbolic secant)	120
⠭⠊ sin (sine)	120
⠭⠊⠓ sinh (hyperbolic sine)	120
⠭⠏ sup (supremum)	120

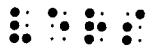
20 ⠮ (dots 2-3-4-5)

⠮ English t	22
German teh	23
Greek tau	24
Hebrew teth	24
Russian teh	25
⠮⠞ tan (tangent)	120
⠮⠞⠓ tanh (hyperbolic tangent)	120

21 ⠳ (dots 1-3-6)

⠳ English u	22
German oo	23
Greek upsilon	24
Russian oo	25

22  (dots 1-2-3-6)



23  (dots 1-3-4-6)

24  (dots 1-3-4-5-6)

25  (dots 1-3-5-6)

25 ⠒ (Cont.)

		Page
Hebrew zayin		23
Russian zeh		25

26 ⠶ (dots 1-2-3-4-6)

curved left full arrowhead	❨	146
factorial	!	152
Greek chi		24

27 ⠿ (dots 1-2-3-4-5-6)

blunted left full arrowhead	⟦	146
blunted right full arrowhead	⟧	146
general omission symbol		70

28 ⠷ (dots 1-2-3-5-6)

left parenthesis	(	122

29 ⠦ (dots 2-3-4-6)

		26
Greek stigma		23
Hebrew tsadi		152
single integral	∫	25
Russian yerih		
double integral	∫∫	152
triple integral	∫∫∫	153

				Page
29	⠨	(Cont.)		
	⠨⠐	integral with superposed circle	$\oint$	153
	⠨⠸	integral with superposed rectangle	$\oint\!\!\!\!\square$	153
	⠨⠬	integral with superposed square	$\oint\!\!\!\!\square$	153
	⠨⠿	integral with superposed infinity	$\oint\!\!\!\!\infty$	153
30	⠾	(dots 2-3-4-5-6)		
	⠾	right parenthesis	)	122
31	⠐	(dots 1-6)		
	⠐	dot, and times	·	128, 129
		Hebrew chaph		23
	⠐⠣	dot within inclusion sign	$\Subset$	140
	⠐⠜	dot within reverse inclusion sign	$\Supset$	140
	⠐⠶	dot between bars of equals sign	$\doteq$	140
32	⠰	(dots 1-2-6)		
	⠰	directly-over indicator (first order)		97
		index-of-radical indicator		108
		makes nearer arrowhead point up		145
	⠰⠰	directly-over indicator (second order)		97
	⠰⠰⠒	upper limit	$\overline{\lim}$	119
	⠰⠨	upper integral	$\overline{\int}$	153

33 ⠶ (dots 1-4-6)

		Page
⠶	directly-under indicator (first order)	97
	makes nearer arrowhead point down	145
⠶⠆⠒⠶	lower limit $\underline{\lim}$	120
⠶⠒⠶	lower integral $\int\!\!\!\!\!\!\_$	153
⠶⠶	directly-under indicator (second order)	97

34 ⠷ (dots 1-4-5-6)

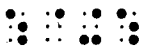
⠷	Greek theta	23
	Hebrew thav	24
	opening simple-fraction indicator	75

35 ⠩ (dots 1-5-6)

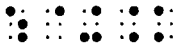
⠩	Greek eta	23
	Hebrew sin	24
	horizontal bar (macron) —	98
	Russian shah	25
⠩⠒⠶	bar over logical product $\overline{\wedge}$	138
⠩⠒⠶⠩	bar over and bar under logical product $\underline{\overline{\wedge}}$	138
⠩⠒⠶⠒⠩	bar over and equals sign under logical product $\underline{\overline{\wedge}}$	138
⠩⠒⠩	bar over single tilde $\overline{\sim}$	139
⠩⠒⠩⠒⠩	bar over double tilde $\overline{\approx}$	139

35 (Cont.) Page

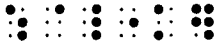
bar over logical sum $\overline{\vee}$ 139



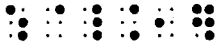
bar over and bar under logical sum $\underline{\overline{\vee}}$ 139



bar over and equals sign under logical sum $\underline{\overline{\vee}}$ 139



bar through inclusion sign $\not\in$ 141

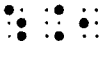


bar through reverse inclusion sign $\not\ni$ 141

bar over inclusion sign $\overline{\subset}$ 138

bar over reverse inclusion sign $\overline{\supset}$ 138

bar over less than sign $\overline{<}$ or $\overline{\leqslant}$ 138



bar over greater than sign $\overline{>}$ or $\overline{\geqslant}$ 137

36 (dots 1-2-4-6)

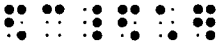
Hebrew ayin 24

Russian yah 25

shape indicator 110

arc concave upward ⌒ 110

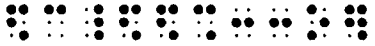
circle ○ 111



circle with interior dot ⊙ 112

circle with interior arrow pointing up ↑ 112

circle with interior arrow pointing up followed by interior arrow pointing down ⇅ 112



circle with interior arrow pointing down ↓ 112

circle with interior arrow pointing down followed by interior arrow pointing up ⇵ 112

36 ⠶ (Cont.)

		Page
⠶⠶⠒⠶⠶⠒⠶⠶⠶	circle with interior arrow pointing left	112
⠶⠶⠒⠶⠶⠒⠶⠶⠶⠒⠶⠶⠒⠶	circle with interior arrow pointing left over interior arrow pointing right	112
⠶⠶⠒⠶⠶⠒⠶⠶⠶	circle with interior arrow pointing right	112
⠶⠶⠒⠶⠶⠒⠶⠶⠶⠒⠶⠶⠒⠶	circle with interior arrow pointing right over interior arrow pointing left	112
⠶⠶⠒⠶⠶⠒⠶	circle with interior plus sign	112
⠶⠶⠒⠶⠶⠒⠶	circle with interior minus sign	112
⠶⠶⠒⠶⠶⠒⠶	circle with interior cross	112
⠶⠶	diamond	111
⠶⠒	ellipse (oval)	111
⠶⠶	parallelogram	111
⠶⠶	rhombus	111
⠶⠶⠶	irregular hexagon	111
⠶⠒	intersecting lines	111
⠶⠒	is parallel to	111
⠶⠒	arrow barbed at right (contracted form)	98, 135
	right-pointing arrow (contracted form)	110
⠶⠶	is perpendicular to	111
⠶⠶⠶	irregular pentagon	111
⠶⠶	quadrilateral	111
⠶⠶	rectangle	111
⠶⠒	star	52, 111, 130

36 ⠤ (Cont.)		Page
⠣⠄	regular triangle (equilateral)	112
⠣⠤⠜	acute triangle	113
⠣⠤⠴	isosceles triangle	113
⠣⠤⠼	obtuse triangle	113
⠣⠤⠾	right triangle	114
⠣⠤⠷	scalene triangle	114
⠣⠄	trapezoid	111
⠣⠂⠒	arrow dotted at left (no barb)	98
⠣⠂⠒⠕	arrow dotted at left and barbed at right	98
⠣⠂⠒⠂	arrow dotted at both ends	98
⠣⠆⠂⠒⠒⠆	vertical two-way arrow	135
⠣⠆⠒⠒⠂	arrow pointing up	135
⠣⠆⠒⠒⠂⠐⠣⠒⠒⠆	arrow pointing up followed by arrow pointing down	140
⠣⠆⠒⠒⠂⠐⠣⠶⠒⠆	arrow pointing up followed by boldface arrow pointing down	140
⠣⠶⠒⠒⠆⠐⠣⠒⠒⠆	boldface arrow pointing up followed by arrow pointing down	140
⠣⠶⠒⠒⠆⠐⠣⠶⠒⠆	boldface arrow pointing up followed by boldface arrow pointing down	140
⠣⠂⠒⠒⠂	arrow pointing down	135
⠣⠒⠒⠂⠐⠣⠒⠒⠆	arrow pointing down followed by arrow pointing up	140
⠣⠶⠒⠒⠂⠐⠣⠒⠒⠆	arrow pointing down followed by boldface arrow poinitng up	140
⠣⠶⠒⠒⠂⠐⠣⠶⠒⠆	boldface arrow pointing down followed by arrow pointing up	140

36 ⠶ (Cont.)

		Page
⠶⠶⠒⠒⠶⠒⠶⠶⠒⠶⠒⠶⠶⠶	boldface arrow pointing down followed by boldface arrow pointing up	140
⠶⠒⠶	angle	110
⠶⠒⠶⠒⠶⠶	arrow barbed at left	98
	left-pointing arrow	110
⠶⠒⠶⠶⠶	arrow barbed at both ends	98
	horizontal two-way arrow	135
⠶⠒⠶⠶⠶	arrow barbed at left and dotted at right	98
⠶⠒⠶⠶⠶⠒⠶⠶⠒	arrow pointing left over boldface arrow pointing right	137
⠶⠒⠶⠶⠒⠶	arrow with hollow dot at right and barbed at left	98
⠶⠒⠶⠒⠶⠶	alternate exterior angles	113
⠶⠒⠶⠒⠶⠶	alternate interior angles	113
⠶⠒⠶⠶⠶⠶	complementary angles	113
⠶⠒⠶⠶⠶	corresponding angles	113
⠶⠒⠶⠒⠶⠶	exterior angles	113
⠶⠒⠶⠒⠶⠶	interior angles	113
⠶⠒⠶⠶⠶⠶	adjacent angles	113
⠶⠒⠶⠒⠶⠶	obtuse angle	113
⠶⠒⠶⠶⠶⠶	right angle	113
⠶⠒⠶⠒⠶⠶	straight angle	113
⠶⠒⠶⠶⠶⠶	supplementary angles	113
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⠶⠒⠒⠶	arrow barbed at right (uncontracted form)	98
⠶⠒⠒⠶⠖⠒⠒⠒	arrow pointing right over arrow pointing left	137
⠶⠒⠒⠶⠖⠒⠒⠒	arrow pointing right over boldface arrow pointing left	137
⠶⠒⠒⠒	arrow dotted at right (no barb)	98
⠶⠒⠒⠒⠖⠒⠒⠒	long arrow pointing right over short arrow pointing left	137
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⠶⠒⠒⠒⠒	arrow with hollow dot at right (no barb)	98
⠶⠒⠶	square	111
⠶⠒⠳⠶⠒⠶	square with interior dot	113
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⠶⠳⠶⠶⠶⠶⠳⠶⠶⠹	boldface arrow pointing left over boldface arrow pointing right	137
⠶⠳⠶⠶⠹⠶⠳⠶⠶	boldface arrow pointing right over arrow pointing left	137
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⠶⠳⠶	filled-in square	130
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⠶⠳⠶⠶⠶⠹	arrow with hollow dot at left and barbed at right	98
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⠶	closing cancellation indicator	73
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38 ⠹ (dots 1-2-5-6)

⠹	Russian yu	25
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	vertical bar as a sign of grouping	123
	vertical bar (is a factor, divides)	130
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Page

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vertical bar through shaft of arrow pointing right  141

39 ⠧ (dots 2-4-6)

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40 ⠧⠦ (dots 2-4-5-6)

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	curved division signs on left and right, separation line above	⎫‾⎧	160
			
	curved division sign on right, separation line above	‾⎧	160
			
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⠒⠤⠒ ⠒⠤⠒ ⠒⠤⠒ ⠒⠤⠒	straight or slant division sign on right, separation line above	160

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			Page
⠒⠤⠲	numeral 4	4	7
	period	.	42

45 ⠒⠂ (dots 2-6)

			Page
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			Page
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47 ⠒⠿ (dots 2-3-5-6

			Page
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⠒⠿ ⠒⠿	ordinary double arrow shaft	=	145

				Page
47	⠸⠸	(Cont.)		
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	⠸⠸ ⠸⠸ ⠸⠸ ⠸⠸ ⠸⠸		carried-number indicator for addition (varying in length)	161
48	⠸⠇	(dots 2-3-6)		
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49	⠸⠩	(dots 3-5)		
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51	⠸⠹	(dots 3-4)		
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52		(dots 3-4-6)		
			regular plus	+ 129
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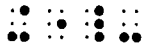
			regular plus followed by regular minus	+— 130
			regular plus followed by boldface minus	+— 130
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			closing simple-fraction indicator	75
			numeric indicator	7
54		(dots 3-4-5)		
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55		(dot 3)		
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56		(dots 3-6)		
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⠶⠐⠶	regular minus followed by regular plus —+	129
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⠐⠐	at @	152
⠐⠶	cent ¢	152
⠐⠲	partial derivative (round d) ∂	153
⠐⠢	membership (is an element of) ε or ∈ or ∈	135
⠐⠖	crossed h ħ	152
⠐⠔	pound sterling £	153
⠐⠬	barbed right upper arrowhead ↘	146
⠐⠎	dollar sign $	152
⠐⠒	curved right upper arrowhead ↷	146
⠐⠣	curved left upper arrowhead ↶	146
	universal quantifier (for all, for each, for every) ∀ or ∀	153
⠐⠤	blunted left upper arrowhead ⌐	146
	blunted right upper arrowhead ¬	146
	existential quantifier (there exists, for some) ∃ or ∃	153
⠐⠿⠐	existential quantifier (there exists uniquely, for exactly one) ∃| or ∃|	153

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⠷⠜	right square bracket	]	122
⠷⠡	cross (Cartesian product, multiplication sign)	×	137
⠷⠣⠷⠜	equivalence	⇔	129
⠷⠯	logical product (and, meet)	∧	129
⠷⠯⠳	bar under logical product	∧̲	138
⠷⠯⠷⠳	equals sign under logical product	≙	138
⠷⠔	simple tilde (is related to, is similar)	~	136
⠷⠔⠳	bar under single tilde	≃	139
⠷⠔⠷⠔	double tilde	≈	139
⠷⠔⠷⠔⠳	bar under double tilde	≋	139
⠷⠔⠷⠔⠷⠔	equals sign under double tilde	≋	139
⠷⠔⠷⠳	equals sign under single tilde	≃	139
⠷⠙	crossed d	đ	152
⠷⠿	general reference indicator		52
⠷⠩	straight left upper arrowhead	⌊	146
	straight right upper arrowhead	⌋	146
⠷⠡	barbed left upper arrowhead	↙	146
⠷⠐	reverse membership (contains the element)	∍ or ∋ or ∋	136
⠷⠴	percent sign	%	153
⠷⠷	since (because)	∵	153

57 ⠷ (Cont.)

		Page
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⠷⠾⠸	bar under logical sum $\underline{\vee}$	139
⠷⠾⠷⠸	equals sign under logical sum $\underline{\underline{\vee}}$	139
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⠷⠷⠷	lower-case script Russian-letter indicator	208
⠷⠷⠄	capital Russian-letter indicator	208
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⠷⠾⠸	upper left half-bracket ⌜	123
⠷⠾⠼	upper right half-bracket ⌝	123
⠷⠾⠄⠸	upper-left enlarged half bracket ⌜	123
⠷⠾⠄⠼	upper-right enlarged half bracket ⌝	123
⠷⠾	lower-case script German-letter indicator	208
⠷⠾⠸	left barred bracket ⟦	123
⠷⠾⠼	right barred bracket ⟧	123
⠷⠾⠄	capital script German-letter indicator	208
⠷⠾⠄⠸	left enlarged barred bracket ⟦	123
⠷⠾⠄⠼	right enlarged barred bracket ⟧	123
⠷⠾	lower-case script Greek-letter indicator	208
⠷⠾⠼	crossed lambda ƛ	152

57	 (Cont.)		Page
		capital script Greek-letter indicator	208
		lower-case script English-letter indicator	208
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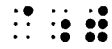
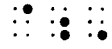
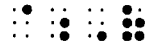
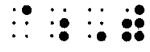
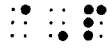
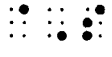
		lower right half-bracket ⌋	123
		capital script English-letter indicator	208
		lower-left enlarged half-bracket ⌊	123
		lower-right enlarged half-bracket ⌋	123
		angstrom unit Å	152
		paragraph mark ¶	52, 129
		crossed R ℟	152
		single section mark §	52, 130
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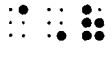
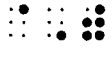
		enlarged left square bracket [	122
		enlarged right square bracket]	122
		extended tilde ~	136
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		superscript with superscript with superscript indicator	82

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⠸⠸⠸	superscript with superscript with subscript indicator	82
⠸⠸⠸	superscript with subscript indicator	82
⠸⠸⠸	superscript with subscript with superscript indicator	82
⠸⠸⠸	superscript with subscript with subscript indicator	82

59 ⠸ (dots 4-5-6)			
⠸	boldface-type indicator		36
	filled-in shape indicator		110
	German-letter indicator		21
	punctuation indicator		110
	tally mark	|	153
	vertical line used in division arrangements (varying in length)	|	161
⠸⠸	identify (is congruent to, is identical with)	≡	135
⠸⠸	ampersand (and, logical product)	&	128
⠸⠸	variation (varies as)	∝	136
⠸⠸	back slash (divides, is a factor of)	\	128
⠸⠸	caret (circumflex)	^	99, 152
⠸⠸	inverted caret	v	99
⠸⠸	opening fractional-part-of-mixed-number indicator		75
⠸⠸	interior shape-modification indicator		110
⠸⠸	single dagger	†	52, 128

59 ⠸ (Cont.)

Page

⠸⠂ boldface single vertical bar | 123

boldface vertical bar (end of proof) | 153

⠸⠂⠸⠂ boldface double vertical bar || 123

⠸ straight or slant division sign on left, separation line below ⌊ or ⧹ 161

⠶⠶⠶

⠸ ⠸ straight division signs on left and right, separation line below ⌊⌋ 161

⠶⠶⠶

⠸ straight or slant division sign on right, separation line below ⌋ or ⧸ 161

⠶⠶⠶

⠸ synthetic division with straight line on left, separation line below ⌊ 162

⠸

⠶⠶⠶

⠸ synthetic division with straight line on right, separation line below ⌋ 162

⠸

⠶⠶⠶

⠸⠄ question mark (as a modifier) ? 99

⠸⠄ empty set (represented by zero with vertical or oblique bar through it) ϕ or ∅ 152

⠸⠌ diagonal line or slash / 75

diagonal fraction line / 128

59 ⠻ (Cont.)

		Page
⠻⠖	boldface plus +	129
⠻⠖⠐⠖	boldface plus followed by regular minus +—	129
⠻⠖⠐⠻⠖	boldface plus followed by boldface minus +—	129
⠻⠻	closing fractional-part-of-mixed-number indicator	75
⠻⠤	boldface minus —	129
⠻⠤⠐⠖	boldface minus followed by regular plus —+	129
⠻⠤⠐⠻⠖	boldface minus followed by boldface plus —+	129
⠻⠣	boldface left square bracket [	122
⠻⠜	boldface right square bracket]	122
⠻⠁	lower-case boldface Russian-letter indicator	208
⠻⠠⠁	capital boldface Russian-letter indicator	208
⠻⠫	lower-case boldface German-letter indicator	208
⠻⠳	double dagger ‡	52
⠻⠠⠫	capital boldface German-letter indicator	208
⠻⠈	inclusion sign (is contained in, is a subset of) ⊂	135
⠻⠈⠸	bar under inclusion sign (is a subset of) ⊆	138
⠻⠈⠐⠶	inclusion sign through equals sign ⫅	141
⠻⠈⠐⠶	equals sign under inclusion sign (is a subset of) ⊆	138
⠻⠛	lower-case boldface Greek-letter indicator	208
⠻⠶	boldface equals sign =	135
⠻⠕	reverse inclusion sign (contains, implies) ⊃	136

			Page
59	⠻ (Cont.)		
	⠻⠒⠩⠹	bar under reverse inclusion sign ⊇	139
	⠻⠒⠩⠒⠩⠭	reverse inclusion sign through equals sign ⫐	141
	⠻⠒⠩⠒⠭	equals sign under reverse inclusion sign ⊇	139
	⠻⠒⠒	capital boldface Greek-letter indicator	208
	⠻⠂	lower-case boldface English-letter indicator	208
	⠻⠂⠒	capital boldface English-letter indicator	208
	⠻⠆	capital German-letter indicator	208
	⠻⠐⠒	closing boldface type indicator for words, phrases, and mathematical statements	36
60	⠼ (dot 5)		
	⠼	base-line indicator	82
		multipurpose indicator	158
	⠼⠣	less than sign (regular) <	135
	⠼⠣⠹	bar under less than sign (is less than or equal to) ≤ or ⩽	138
	⠼⠣⠣⠼⠣⠣	nest of two less than signs with straight sides (is small compared with) ≪	141
	⠼⠣⠒⠼⠣⠒⠼⠣	less than sign followed by equals sign followed by greater than sign < = >	140
	⠼⠣⠒⠼⠜	less than sign followed by greater than sign < >	140
	⠼⠣⠒⠣⠒	equals sign under less than sign (is less than or equal to) ≤	138
	⠼⠣⠈⠼⠣⠈	caret over horizontal bar ^	136
	⠼⠣⠤⠼⠣⠤	dot under horizontal bar .	137
	⠼⠣⠈⠤⠼⠣⠈	caret under horizontal bar (is perspective to, perspective correspondence) ^	138
	⠼⠣⠒⠣	ratio sign (is to) :	135

60 ⠸⠼ (Cont.)		Page
⠸⠼⠐⠶	dot under simple tilde	137
⠸⠼⠐⠒	dot over equals sign (is approximately equal to)	136
⠸⠼⠘⠒	equilateral triangle over equals sign	136
⠸⠼⠳⠒	vertical bar over equals sign	136
⠸⠼⠈⠒	caret over equals sign	136
⠸⠼⠬⠒	inverted caret over equals sign	136
⠸⠼⠢⠒	question mark over equals sign	136
⠸⠼⠴⠒	degree sign over equals sign (is equal in degrees to)	136
⠸⠼⠣⠒	left-pointing caret over equals sign	136
⠸⠼⠜⠒	right-pointing caret over equals sign	136
⠸⠼⠒⠒	two dots over and two dots under equals sign	136
⠸⠼⠤⠒	dot over and dot under equals sign	136
⠸⠼⠒⠈	caret under equals sign (is projective to, projective correspondence)	136

61 ⠸⠼ (dots 4-6)		
⠸⠼	decimal point (American and Continental)	7
	first inner radical indicator	108
	Greek-letter indicator for standard letters	22
	Greek-letter indicator	22
	italic-type indicator	36
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	structural shape-modification indicator	110

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⠸⠒⠒⠘⠬	equals sign over logical product	138
⠸⠒⠒⠘⠬⠸⠤	equals sign over and bar under logical product	138
⠸⠒⠒⠘⠬⠸⠒⠒	equals sign over and equals sign under logical product	138
⠸⠒⠒⠘⠔	equals sign over single tilde	139
⠸⠒⠒⠘⠔⠔	equals sign over double tilde	139
⠸⠒⠒⠘⠌	equals sign over logical sum	139
⠸⠒⠒⠘⠌⠸⠤	equals sign over and bar under logical sum	139
⠸⠒⠒⠘⠌⠸⠒⠒	equals sign over and equals sign under logical sum	139
⠸⠒⠒⠘⠣⠨⠅	equals sign with superposed inclusion sign	141
⠸⠒⠒⠘⠜⠨⠅	equals sign with superposed reverse inclusion sign	141
⠸⠒⠒⠘⠣⠂	equals sign over inclusion sign (is a subset of)	141
⠸⠒⠒⠘⠜⠂	equals sign over reverse inclusion sign	139
⠸⠒⠒⠘⠣	equals sign over less than (is equal to or less than)	138
⠸⠒⠒⠘⠜	equals sign over greater than (is equal to or greater than)	137
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